Formalisation and analysis of Dalvik bytecode

Erik Ramsgaard Wognsen *, Henrik Søndberg Karlsen *, Mads Chr. Olesen *, René Rydhof Hansen *

Department of Computer Science, Aalborg University, Denmark

HIGHLIGHTS

- We present a study of language features used in 1700 Android apps.
- We formalise the semantics of the Dalvik bytecode language.
- We formally specify a control flow analysis for the Dalvik bytecode language.
- We also formalise reflection both in semantics and analysis.
- A prototype implementation of the analysis is discussed.

ARTICLE INFO

Article history:
Received 1 August 2012
Received in revised form 1 November 2013
Accepted 17 November 2013
Available online 1 December 2013

Keywords:
Dalvik
Android
Static analysis
Flow logic
Reflection

ABSTRACT

With the large, and rapidly increasing, number of smartphones based on the Android platform, combined with the open nature of the platform that allows “apps” to be downloaded and executed on the smartphone, misbehaving and malicious (malware) apps are set to become a serious problem. To counter this problem, automated tools for analysing and verifying apps are essential. Furthermore, to ensure high-fidelity of such tools, it is essential to formally specify both semantics and analyses.

In this paper we present, to the best of our knowledge, the first formalisation of the complete Dalvik bytecode language including reflection features and the first formally specified control flow analysis for the language, including advanced control flow features such as dynamic dispatch, exceptions, and reflection. To determine which features to include in the formalisation and analysis, 1700 Android apps from the Google Play app market (formerly known as Android Market) were downloaded and examined.

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1. Introduction

Since Android was first introduced, more than 100 million Android devices have been activated, and more than 400,000 new devices are activated every day. This makes Android one of the most widespread and fastest growing computing platforms for smartphones and tablet computers. The combination of the wide distribution and the open nature of the Android platform, where apps can be downloaded and installed not only from the official Google Play app market¹ but also from unknown, untrusted, and potentially malicious third parties makes it obvious that tools are needed to ensure, and possibly certify, that apps are well-behaved and do not access information or functionality not explicitly allowed and intended by the user. The problem is further exacerbated by the often sensitive and private nature of information stored on

¹ This work is based on the paper “Study, Formalisation, and Analysis of Dalvik Bytecode” presented at the Seventh Workshop on Bytecode Semantics, Verification, Analysis and Transformation (BYTECODE 2012) [14] and the master’s thesis of the first two authors [13].

* Corresponding authors.

E-mail addresses: erw@cs.aau.dk (E.R. Wognsen), henrik.karlsen@gmail.com (H.S. Karlsen), mchro@cs.aau.dk (M.C. Olesen), rrh@cs.aau.dk (R.R. Hansen).

a smartphone as well as the potential for apps to (ab-)use services that cost the user money, e.g., by secretly sending text messages to expensive premium numbers [3].

In order to develop trustworthy tools for analysis and especially for certification, we believe it is necessary to have a formal underpinning of the target platform allowing sound analyses to be developed with confidence. In this paper we first present a study of 1700 Android apps, carried out in order to determine what Dalvik instructions and language features are most often used in typical apps. Based on the results of this study, we develop a formal operational semantics for the Dalvik bytecode language [1]. Dalvik bytecode is similar to Java bytecode. The two most notable differences are: Dalvik is register based rather than stack based and the local registers are untyped. These differences are reflected in their two instruction sets. In addition, there are a number of differences in their respective runtime systems, which we will not discuss any further here. Based on the operational semantics, we develop and formally specify a control flow analysis for the Dalvik bytecode language. The analysis is intended both as the basis for further and more specialised analyses, but also by itself for detecting potentially malicious actions, e.g., leaking private information or surreptitiously calling expensive phone numbers.

To the best of our knowledge, this is the first such formalisation of the Dalvik bytecode language and an accompanying control flow analysis. Since our study revealed that more than half of the apps examined use reflection, we also formalise the semantics of the reflection API and also extend the control flow analysis to cover many uses of reflection as determined by the study.

Finally, we have developed a prototype implementation of the control flow analysis in order to investigate properties and behaviour of the analysis (Section 6). The prototype is strictly meant as a proof-of-concept and not a finished production tool. Consequently the implementation has some notable limitations regarding the analysis of real apps: there is no support for libraries and APIs, limited support for reflection, and the exception analysis, although formalised, is not implemented either. Even with these limitations the prototype has been used to perform a number of useful analyses described in Sections 7 and 8.

While Android apps are generally developed in Java, compiled to Java bytecode, and only then converted to Dalvik bytecode, we focus here on Dalvik bytecode because it is the common executable format for all Android apps and, therefore, offers the best opportunity for performing analyses as close to the code actually executed as possible and allows us to sidestep issues relating to decompiling and reverse engineering apps, cf. [7].

1.1. Related work

In [4] the tool ComDroid is described as a tool that performs “flow sensitive, intra-procedural static analysis with limited inter-procedural analysis” of Dalvik bytecode programs. It is designed to analyse the communication between Android applications through the so-called Intents, the Android equivalent of events, and to find potential security vulnerabilities in the communication patterns of applications. The same ComDroid tool is used as a component in another analysis tool called Stowaway, that analyses API calls in applications to determine if they are over-privileged [8]. In order to improve the precision and efficacy of the analysis, Stowaway incorporates some analysis of the reflection features found in Dalvik bytecode (through the java.lang.reflect library). Both ComDroid and Stowaway are sophisticated analysis tools covering not only the Dalvik bytecode language but also important parts of the API and the Android platform itself. However, since the analyses are not actually specified in detail, neither formally nor informally, it is impossible to evaluate their exact strengths and weaknesses. Indeed, it is stated in [8] that Stowaway makes a “primarily linear traversal” and that it “experiences problems with non-linear control flow”. This emphasises the need for a formalisation of both the Dalvik bytecode language as well as the control flow analysis.

Another approach to analysing Android applications for potential security violations is discussed in [7]. Here Android applications are analysed by first recovering the Java source through decompilation and then using the Fortify SCA static analysis tool to detect potential security vulnerabilities. While the paper reports impressive results using this approach, it is also noted that it was not possible to recover the source code for all the targeted applications and thus making analysis of those applications impossible. Analysing directly at the bytecode level sidesteps this issue.

The approach described in [24] takes advantage of the fact that most, if not all, Android applications are developed in Java and adapts the Julia framework (see [28]) for Java bytecode analysis to the specificities of the Java bytecode that results from developing Android applications (before being converted to Dalvik bytecode). As an example, the Julia-based analysis can handle the specific use of reflection for XML specified graphical views prevalent on the Android platform. It does not handle the general case of reflection nor the special instances handled by our approach. The Julia framework is theoretically sound, comprehensive, and well-documented, but the described solution requires access to the Java bytecode version of an application in order to analyse it.

In [12] the authors formalise a simple language, μ-Dalvik, and translate Dalvik bytecode into μ-Dalvik, with the goal of performing symbolic execution of Dalvik programs. μ-Dalvik corresponds roughly to the core of the language that we have formalised, but with some important differences: Dalvik bytecode might be mapped to several μ-Dalvik instructions, whereas our mapping is always one to one. In addition we have formalised exceptions, the array-length instruction,
and some common cases of reflection, neither of which are formalised in $\mu$-Dalvik. As our study shows, reflection is used by a majority of apps making it a crucial feature to model.

Much of the work on Dalvik bytecode has been inspired by similar work on Java bytecode, including the current paper which borrows the main methodology from [11]. One of the first, possibly the first, formal semantics for Java bytecode language was defined by Bertelsen in [2]. Other early work on formalisation and analysis of Java bytecode includes [10,9] where a core bytecode language is given a formal semantics and a type system for analysis is proven correct. Spoto provided one of the first formal semantics used as the basis for abstract interpretation of Java bytecode [29]. This work inspired the Julia framework for analysis [28] which is based on traditional abstract interpretation parameterised with different abstract domains. As mentioned above the Julia framework has recently been adapted to cover Dalvik bytecode [24] translated from Java bytecode. One of the earliest and most comprehensive tools for Java bytecode analysis is the Soot framework [30]. The work on Soot has focused more on effectiveness and efficiency than on formalisation and thus lacks formal specification and proof.

2. Study of apps

To identify which Dalvik bytecode instructions and which Java language features are used in typical Android apps, we collected and examined the 50 most popular free apps of each category of Android Market (now part of Google Play), 1700 apps in total. Notable features include code obfuscation, threading, reflection, native libraries, and dynamic class loading. The apps were collected in November 2011 using Android 2.3.3 on a Samsung Nexus S smartphone.

For efficiency reasons the Dalvik bytecode language contains several specialised variants of many common instructions, e.g., there are numerous variants of the move instruction. For our study we have grouped instruction variants that are semantically similar, e.g., most variants of the move instruction belong to the same group. In the semantics (see Section 3) we use the same notion of grouping to abstract and generalise the original 218 Dalvik bytecode instructions into a set of 39 instructions. The mapping between the original and generalised instructions is given in Appendix A.

In our study we found that, with the exception of the filled-new-array instruction, all types of Dalvik bytecode instructions are used in more than half of the studied apps. In particular, the instructions invoke-direct and return-void are used in every app and even the two most rare instructions, sparse-switch and filled-new-array, are used in 69.7% and 22.3% of the studied apps, respectively. The instructions that occur most frequently are invoke-virtual and move-result, which are used more than 12 million times each in total in the 1700 apps. In comparison, filled-new-array is used 1930 times. For full details, see Appendix B.

The observations made from studying the use of Java features are summarised in Table 1 and are explained in detail below. For the study we have separated code into developer code and library code. Developer code is code that lies within the natural packages for the application. For an application company.app this means all classes located directly in the packages /, /company/, /company/app/, and any subpackages in /company/app/. Library code is everything else.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Used by apps</th>
<th>Hereof in libraries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obfuscated source</td>
<td>64.82%</td>
<td>–</td>
</tr>
<tr>
<td>Has native libraries</td>
<td>20.35%</td>
<td>–</td>
</tr>
<tr>
<td>java/lang/Thread</td>
<td>90.38%</td>
<td>24.07%</td>
</tr>
<tr>
<td>java/lang/ClassLoader</td>
<td>73.00%</td>
<td>55.92%</td>
</tr>
<tr>
<td>java/lang/ClassLoader</td>
<td>39.71%</td>
<td>81.19%</td>
</tr>
<tr>
<td>java/lang/Runtime;-&gt;exec</td>
<td>19.53%</td>
<td>80.44%</td>
</tr>
</tbody>
</table>

Code obfuscation, especially using ProGuard, is used to a large extent. We searched for classes named “a” within apps in the data set, and used this as an approximation to determine if an app contains any obfuscated code. The same approach was used in [7] which found 36% of apps to include obfuscated code. We found the class in 64.82% of the apps. Obfuscation is recommended by Google, but makes it harder to manually inspect the code.

Native libraries, i.e., ARM shared object (.so) files, were included in 20.35% of the apps we studied. A previous study [7] found that, of their 1,100 studied apps from September 2010, only 6.45% included shared objects. We presume the increased usage is because the Android NDK, released June 25, 2009, has gained more widespread use in 2011.

Threading, as indicated by the use of monitors, i.e., the Java synchronized keyword, was found in 88% of the apps. Furthermore, 90.18% of the apps include a reference to java/lang/Thread. These observations are not conclusive, but indicate that multi-threaded programming is wide-spread. However, further studies are needed to substantiate the results.

5 In addition, 15 apps included the ARM executable gdbserver.
6 Previously “Native Development Kit”, a toolset that facilitates interfacing Java/Dalvik with C/C++ and native (ARM) code.
The Java method `Runtime.exec()` is used to execute programs in a separate native process and is present in 19.53% of the apps. We manually inspected some of these uses. Most of them do not use a hardcoded string as the argument to `exec()`, but of those that do, we found execution of both the `su` and `logcat` programs which, if successful, gives the app access to run programs as the super user on the platform or read logs (with private data [7]) from all applications, respectively. Some apps also use the `pm install` command to install other apps at runtime.

Class loading Of the studied apps 39.71% contain a reference to the Java class loader library, `java/lang/ClassLoader`, or a subclass such as the `dalvik/system/DexClassLoader`. However, only 13.1% of apps use the `loadClass()` or `defineClass()` methods to actually load or define classes at runtime. Class loading allows the loading of Dalvik executable (DEX) files and JAR files while class definition allows for programmatic definition of Java classes, e.g., from scripting languages such as Javascript. If the classes being loaded are not present, e.g., if they are downloaded from the Internet, the app cannot be analysed statically before installation. Furthermore, if the classes being loaded are created dynamically from other languages, analysing the use before installation would require the analysis tool to parse/analyse these languages. A simpler solution for handling the apps that use these features would be to analyse the class just before it is being loaded, on the device. However, we consider this as out of scope for this paper.

Class Transformation allows developers to change behaviour of classes at runtime, before they are loaded by the VM [22]. It is a Java feature, and is therefore also available in Android. The transformations allowed include adding new instructions and changing control flow. We found no apps in our data set that use this feature, and will therefore not return to this subject.

Reflection has been reported to be used extensively in Android apps for accessing private and hidden classes, methods, and fields, for JSON[7] and XML parsing, and for backward compatibility [8]. We confirmed these observations by manual inspection. Of the 940 apps studied in [8], 61% were found to use reflection, and using automated static analysis they were able to resolve the targets for 59% of the reflective calls.

73% of the apps in our data set use reflection. This indicates that a formalisation of reflection in Dalvik is necessary to precisely analyse most apps. Reflection resolves classes, methods, and fields from strings. When these are statically known, static analysis becomes possible. We treat this in Section 5.

Javascript Interfaces allow Javascript in a webpage embedded in an app to control that app. Android supports in-app loading of webpages, through the WebKit API[9] that provides a custom embedded web browser. This API includes the `addJavascriptInterface()` method whose purpose is to make the methods on a Java object available to Javascript code. The method is used in 39% of the apps in our data set. The interface allows webpages loaded by the app to call methods on the Java object. Previous studies have shown that advertisement and analytics libraries use this to give the third-party advertisement companies access to sensor information, such as location updates [18]. We confirmed this use through manual inspection, and furthermore discovered apps that were practically webpages, and where the Dalvik code merely loads the page and extends the browser functionality, e.g. by allowing the webpage to send text messages from the phone.

3. Operational semantics

In this section we describe the formalisation of the Dalvik bytecode language using structural operational semantics [25]. With the exception of instructions related to concurrency, we have formalised the full generalised instruction set of 39 instructions, reduced from the full Dalvik instruction set comprising 218 instructions, see Appendix A. Below we present the semantic rules for a few representative instructions and refer to [13] for an exhaustive list. The approach is inspired by a similar effort to formalising the Java Card bytecode language [27,11].

To simplify our work, we have made three convenient, but minor, generalisations: simplification of the type hierarchy to avoid dealing with bit-level operations except when absolutely necessary; “inlining” of the constant pools for easier and more direct reference of strings, types, methods, and fields; and finally idealising the program counter by abstracting away the length of instructions. While none of these modifications change the expressive power of a Dalvik application, they greatly simplify presentation and formalisation.

The study described in Section 2 impacted the formalisation in two major ways: it was clear that all of the core bytecode language had to be formalised and also that the reflection API had to be formalised. In order to ensure that the formalisation correctly represents the Dalvik (informal) semantics, we based the formalisation on the documentation for Dalvik [1], inspection of the source code for the Dalvik VM in Android,10 tests of handwritten bytecode, and experiments with disassembly of compiled Java code.

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7 JavaScript Object Notation, a standard data interchange format similar to XML.
8 This number counts direct uses of reflection in the app, not indirect uses through Android APIs that themselves employ reflection such as `Activity.setContentContentView()`.
3.1. Program structure

To facilitate the development of the formal semantics for Dalvik bytecode, it is important to have a good formalisation of the formal structure of an app. We follow the general approach of [27, 11] and use domains equipped with accessor-functions: the domain \( D = D_1 \times \cdots \times D_n \) with functions \( f_i : D \rightarrow D_i \) is expressed in a record-like format: \( D = (f_1 : D_1) \times \cdots \times (f_n : D_n) \). The access functions are used in an object-oriented style where, for \( d \in D \), \( f_i(d) \) is written \( d.f_i \) and \( f_i(d, a_1, \ldots, a_m) \) is written \( d.f_i(a_1, \ldots, a_m) \). The notation \( d[f \mapsto x] \) expresses the domain \( d \) where the value of access function \( f \) is updated to \( x \). Similarly the notation \( g[x \mapsto y] \) expresses the function \( g \) where value \( x \) now maps to \( y \). In the following we use the domains for classes, interfaces, fields, and methods to illustrate our approach. The remaining domains are found in Appendix C.

A class is specified with a class name, the app in which the class is defined, the Java package it belongs to, a superclass, as well as sets of implemented interfaces, fields, access flags, implemented methods, and method declarations (for abstract classes):

\[
\text{Class} = (\text{name}: \text{ClassName}) \times (\text{app}: \text{App}) \times (\text{package}: \text{Package}) \times \\
(\text{super}: \text{Class}) \times (\text{implements}: \mathcal{P}(\text{Interface})) \times (\text{fields}: \mathcal{P}(\text{Field})) \times \\
(\text{accessFlags}: \mathcal{P}(\text{AccessFlag})) \times (\text{methods}: \mathcal{P}(\text{Method})) \times \\
(\text{methodDeclarations}: \mathcal{P}(\text{MethodDeclaration}))
\]

We define the superclass of java/lang/Object to be \( \bot \), hence the domain \( \text{Class} \) for superclasses. For name domains like Class\text{Name} we assume an unlimited supply of unique names.

An interface is similar to a class but with three important differences: in place of a superclass it has a set of super-interfaces (\( \text{super}: \mathcal{P}(\text{Interface}) \)), instead of implemented interfaces it symmetrically has its implementing classes (\( \text{implementedBy}: \mathcal{P}(\text{Class}) \)), and, finally, instead of implemented methods it possibly has one method, a class constructor (\( \text{clinit}: \text{Method} \)). Class constructors are needed to initialise static fields on classes and interfaces and are consequently the only methods that can be implemented directly in interfaces.

A field of a class or an interface has a name, the class or interface where it is defined, a type, an indication of whether it is a static field or not, and then its access flags:

\[
\text{Field} = (\text{name}: \text{FieldName}) \times (\text{class}: \text{Class} \cup \text{Interface}) \times \\
(\text{type}: \text{Type}) \times (\text{initialValue}: \text{Prim} \cup \{\text{null}\}) \times \\
(\text{isStatic}: \text{Bool}) \times (\text{accessFlags}: \mathcal{P}(\text{AccessFlag}))
\]

A method signature specifies how a method can be called: the name of the method, the class or interface where it is declared (though not necessarily implemented), as well as the argument and return types:

\[
\text{MethodSignature} = (\text{name}: \text{MethodName}) \times (\text{class}: \text{Class} \cup \text{Interface}) \times \\
(\text{argTypes}: \text{Type}^\ast) \times (\text{returnType}: \text{Type} \cup \{\text{void}\})
\]

Method declarations specify everything about a method except its implementation. They appear in interfaces and abstract classes and besides the method signature specify a kind, a set of access flags, and the checked exceptions the method can throw. The kind of a method can be direct, which is used for non-overridable methods, i.e., constructors and private or final methods, static for static methods (that are not direct), and virtual for normal, overridable methods including methods specified in interfaces. Access flags indicate accessibility and various properties and include public, private, protected, final, and abstract.

\[
\text{MethodDeclaration} = (\text{methodSignature}: \text{MethodSignature}) \times \\
(\text{kind}: \{\text{virtual, static, direct}\}) \times \\
(\text{accessFlags}: \mathcal{P}(\text{AccessFlag})) \times \\
(\text{exceptionTypes}: \mathcal{P}(\text{Class}))
\]

An actual method is a method declaration plus implementation details: a mapping from locations in the method (program counter values) to instructions,\(^1\) the number of registers used for local variables,\(^2\) a set of exception handlers, and a mapping from locations of data tables in the bytecode to the contents of these tables\(^3\):

\[
\text{Method} = (\text{methodDeclaration}: \text{MethodDeclaration}) \times \\
(\text{instructionAt}: \text{PC} \rightarrow \text{Instruction}) \times \\
(\text{numLocals}: \mathbb{N}_0) \times (\text{handlers}: \mathbb{N}_0 \rightarrow \text{ExcHandler}) \times \\
(\text{tableAt}: \text{PC} \rightarrow \text{DataTable})
\]

\(^1\) Instruction is simply the set of generalised instructions as given in Appendix A.

\(^2\) Dalvik uses registers instead of an operand stack.

\(^3\) Data tables consist of hardcoded array data and switch tables.
For methods, i.e., elements from Method, the class found in the method signature of the method declaration specifies the class (or interface, in the case of clinit) where the method is implemented. For convenience we introduce unambiguous shorthands such as m.kind for m.methodDeclaration.kind where m ∈ Method.

3.2. Semantic domains

Having defined the domains for the overall structure of an app, what remains is to define the semantic domains. The most fundamental semantic domain is that for values. In Dalvik values can be either primitive values or references:

\[ \text{Val} = \text{Prim} + \text{Ref} \]

For our purpose, it is sufficient to let primitive values consist only of numbers: \( \text{Prim} = \mathbb{Z} \). References are either the null reference or an abstract heap location that does not need to be defined in further detail\(^{14}\): \( \text{Ref} = \text{Location} \cup \{ \text{null} \} \).

Local registers are formalised as a map from register names to values, with \( \bot \) denoting undefined register contents: \( \text{LocalReg} = (\mathbb{N}_0 \cup \{ \text{retval} \}) \rightarrow \text{Val} \). Note that a special register, the retval register, is used to transfer return values from invoked methods to the invoking methods.\(^{15}\)

For convenience we use two distinct domains to formalise the heap by splitting it into one for static fields and one for dynamic objects and arrays:

\[ \text{StaticHeap} = \text{Field} \rightarrow \text{Val} \quad \text{and} \quad \text{Heap} = \text{Ref} \rightarrow (\text{Object} + \text{Array}) \]

Objects are formalised as a domain with a class and a mapping from (object) fields to values. Furthermore, in anticipation we add an annotation component, called origin, that records the program point at which the object was created. It is important to note that this has no effect on the semantics, i.e., the actual execution of a program, but is only needed to facilitate the formalisation and proof of correctness for the analysis developed in Section 4:

\[ \text{Object} = (\text{class} : \text{Class}) \times (\text{field} : \text{Field} \rightarrow \text{Val}) \times (\text{origin} : \text{Method} \times \text{PC}) \]

For arrays we do not track the creation point, merely the type, length, and valuation:

\[ \text{Array} = (\text{type} : \text{ArrayType}) \times (\text{length} : \mathbb{N}_0) \times (\text{value} : \mathbb{N}_0 \rightarrow \text{Val}) \]

We can then define stack frames to consist of a method and a program counter value, i.e., a uniquely determined program point, and the local registers:

\[ \text{Frame} = \text{Method} \times \text{PC} \times \text{LocalReg} \]

Special frames, exception frames containing the location of its corresponding exception object on the heap and the address\(^{16}\) of the instruction that threw the exception, are introduced for tracking exceptions that are not handled locally:

\[ \text{ExcFrame} = \text{Location} \times \text{Method} \times \text{PC} \]

This leads to the following definition of call stacks as a sequence of frames except that the top frame may be an exception frame representing an as of yet unhandled exception:

\[ \text{CallStack} = (\text{Frame} + \text{ExcFrame}) \times \text{Frame}^* \]

Together the heaps and the callstack comprise a semantic configuration:

\[ \text{Configuration} = \text{StaticHeap} \times \text{Heap} \times \text{CallStack} \]

3.3. Semantic rules

We specify the semantics as a straightforward structural operational semantics where each configuration comprises a static heap, a heap, and a call stack as defined above. To illustrate the semantics, we present the semantic rules for a few central instructions, first the basic move instruction:

\[
\begin{align*}
\text{m.instructionAt}(pc) &= \text{move } v_1 \ v_2 \\
A \vdash (S, H, (m, pc, R) :: SF) &\implies (S, H, (m, pc + 1, R[v_1 \mapsto R(v_2)]) :: SF)
\end{align*}
\]

The function instructionAt is an access function on the Method domain that identifies the instruction at a given location in a specified method. In the new configuration, the static heap, \( S \), and dynamic heap, \( H \), of the top frame are unchanged. Only

\(^{14}\) Dalvik does not support pointer arithmetic so it will suffice to know that we can model an arbitrary number of unique locations.

\(^{15}\) Besides the retval register, Dalvik supports \( 2^{16} \) numbered regular registers that we generalise to \( \mathbb{N}_0 \).

\(^{16}\) In general we refer to a pair consisting of a method and a program counter value (PC = \( \mathbb{N}_0 \)) as an address. In contexts where the method is given, we sometimes use address about the program counter value alone.
the top frame of the call stack is affected: the program counter is incremented, and the register valuation is updated such that the destination register now maps to the value of the source register.

The invoke-virtual instruction is more involved:

\[
m.\text{instructionAt}(pc) = \text{invoke-virtual } v_1...v_n \text{ meth} \\
R(v_1) = \text{locc} \quad \text{locc} \neq \text{null} \quad o = H(\text{locc}) \\
n = \text{arity(meth)} \quad m' = \text{resolveMethod(meth, o.class)} \neq \perp \\
R' = [0 \mapsto \perp, ..., m'.numLocals - 1 \mapsto \perp, m'.numLocals \mapsto R(v_1), ..., m'.numLocals + n - 1 \mapsto R(v_n)] \\
A \vdash (S, H, (m, pc, R) :: SF) \implies (S, H, (m', 0, R') :: (m, pc, R) :: SF)
\]

It receives \( n \) arguments and the signature of the method to invoke. The first argument, \( v_1 \), is a reference to the object on which the method should be invoked (the "this" pointer). The location of the method is resolved using the auxiliary function \( \text{resolveMethod} \) as explained below and the resulting method is put into a new frame on top of the call stack, with the program counter set to \( 0 \). A new set of local registers, \( R' \), is created where the first \( m'.numLocals \) registers are mapped to \( \perp \_\_Val \) such that they are initially undefined. The arguments are then mapped into the next registers. To handle dynamic dispatch, we define the following function to search for a method matching the given method signature in the ancestry of the given class:

\[
\text{resolveMethod}(\text{meth}, \text{cl}) = \begin{cases} 
\perp & \text{if } \text{cl} = \perp \\
\text{m} & \text{if } \text{m} \in \text{cl.methods} \land \text{meth} \_\_\text{m} \\
\text{resolveMethod}(\text{meth}, \text{cl.super}) & \text{otherwise}
\end{cases}
\]

where \( \text{meth} \_\_\text{m} \) is a predicate formalising when a method signature \( \text{meth} \) is compatible with a given method \( \text{m} \in \text{Method} \), i.e. when the names, argument types and return types match.

The return instruction pops off the top frame, advances the program counter, and passes on the return value by updating the \( \text{return} \) register:

\[
m.\text{instructionAt}(pc) = \text{return } v \\
A \vdash (S, H, (m, pc, R) :: SF) \implies (S, H, (m', pc' + 1, R'[\text{return} \mapsto R(v)]) :: SF)
\]

The Dalvik instruction set also contains some specialised instructions, for example \texttt{const-class} which is a shortcut to create a \texttt{java/lang/Class} instance representing a specified class. This instruction is typically used in conjunction with reflection which we treat in Section 5. A \texttt{java/lang/Class} object has a field called \texttt{name} which refers to a \texttt{java/lang/String} object with the name of the class. The instruction therefore creates a \texttt{Class} and a \texttt{String} and sets the \texttt{String} field value to the (character array) name of the given class and maps the field \texttt{name} on the \texttt{Class} to the newly created string:

\[
m.\text{instructionAt}(pc) = \text{const-class } v \text{ cl} \\
\text{findClassObject}(H, \text{cl}) = \perp \\
(H', \text{locc}) = \text{newObject}(H, \text{java/lang/Class}, m, pc) \quad o_c = H'(\text{locc}) \\
(H'', \text{locc}) = \text{newObject}(H', \text{java/lang/String}, m, pc) \quad o_s = H''(\text{locc}) \\
H''' = H''(\text{locc} \mapsto o_c[\text{field} \mapsto o_c[\text{field}[\text{name} \mapsto \text{locc}]], \text{locc} \mapsto o_s[\text{field} \mapsto o_s[\text{field}[\text{value} \mapsto \text{cl.name}]]]) \\
A \vdash (S, H, (m, pc, R) :: SF) \implies (S, H''', (m, pc + 1, R[v \mapsto \text{locc}]) :: SF)
\]

The auxiliary function \( \text{findClassObject} \) searches the heap for a class object representing the given class. It returns the object location if the object exists, and returns \( \perp \) otherwise.

The new objects are created using the auxiliary function \( \text{newObject} \), also used in the semantics of the \texttt{new-instance} instruction, which returns the modified heap along with a fresh reference to the newly created object:

\[
\text{newObject} : \text{Heap} \times \text{Class} \times \text{Method} \times \text{PC} \rightarrow \text{Heap} \times \text{Ref} \\
\text{newObject}(H, \text{cl}, m, pc) = (H', \text{loc})
\]

The produced heap and location have the following properties: \( \text{locc} \notin \text{dom}(H), H' = H[\text{loc} \mapsto o], o \in \text{Object}, o.class = \text{cl}, \) and \( o.\text{origin} = (m, pc) \). Note in particular, that the current program point is recorded in the \texttt{origin} field of the newly created object.

Class objects are unique,\(^\text{17}\) so if a class object with the desired class was already present on the heap, that one is returned instead of creating a new one:

\[
m.\text{instructionAt}(pc) = \text{const-class } v \text{ cl} \\
\text{findClassObject}(H, \text{cl}) = \text{locc} \neq \perp \\
A \vdash (S, H, (m, pc, R) :: SF) \implies (S, H, (m, pc + 1, R[v \mapsto \text{locc}]) :: SF)
\]

\(^\text{17}\) This only holds for classes loaded by the same class loader. Since our analysis is defined only for the standard class loader, this is not a problem here.
3.4. Exceptions

Exceptions can be thrown either explicitly using the `throw` instruction, or by the system in case of a runtime error, such as a null pointer dereference. Both situations can be seen directly in the `throw` instruction whose semantics depends on its argument:

\[
m\text{.instructionAt}(pc) = \text{throw } v
\]

\[
R(v) = \text{loc} \neq \text{null} \quad H(\text{loc}).\text{class} \subseteq \text{Throwable}
\]

\[
A \vdash \langle S, H, \langle m, pc, R \rangle :: SF \rangle \implies \langle S, H, \langle \text{loc}, m, pc \rangle :: \langle m, pc, R \rangle :: SF \rangle
\]

\[
m\text{.instructionAt}(pc) = \text{throw } v
\]

\[
R(v) = \text{null} \quad (H', \text{loc}) = \text{newObject}(H, \text{NullPointerException})
\]

\[
A \vdash \langle S, H, \langle m, pc, R \rangle :: SF \rangle \implies \langle S, H', \langle \text{loc}, m, pc \rangle :: \langle m, pc, R \rangle :: SF \rangle
\]

In both situations an exception frame is pushed on the stack and the next step depends on whether an appropriate exception handler is available in the method. If one is available, the exception frame is discarded and its exception is put in the `retval` register of the current method which also jumps to the program counter of the handler:

\[
cl = H(\text{loc}).\text{class} \quad \text{findHandler}(m, pc, cl) = pc' \neq \bot
\]

\[
A \vdash \langle S, H, \langle \text{loc}, m, pc \rangle :: \langle m, pc, R \rangle :: SF \rangle \implies \langle S, H, \langle \text{loc}, m, pc \rangle :: SF \rangle
\]

If no exception handler is found, the frame of the currently executing method is discarded but the exception frame retained such that the search continues recursively among the handlers of the next method:

\[
cl = H(\text{loc}).\text{class} \quad \text{findHandler}(m, pc, cl) = \bot
\]

\[
A \vdash \langle S, H, \langle \text{loc}, m, pc \rangle :: \langle m, pc, R \rangle :: SF \rangle \implies \langle S, H, \langle \text{loc}, cl, m, pc \rangle :: SF \rangle
\]

An exception handler has a type for the exceptions it may catch, a program counter value pointing to the handler code, and program counter values defining the boundaries of the region covered by the exception handler:

\[
\text{ExcHandler} = (\text{catchType} : \text{Class} \bot) \times (\text{handlerAddr} : \text{PC}) \times (\text{startAddr} : \text{PC}) \times (\text{endAddr} : \text{PC})
\]

The auxiliary function `findHandler` finds the exception handler matching the location in the method and the given exception class, or `\bot` if no appropriate handler is available. It is itself defined using two auxiliary predicates.

The first, `canHandle`, determines whether an exception handler `h` can handle the given exception `cl` for the specific location `(pc)` in the method:

\[
\text{canHandle}(h, pc, cl) \equiv h.\text{startAddr} \leq pc \leq h.\text{endAddr} \land cl \leq h.\text{catchType}
\]

The catch type of a catch-all handler, corresponding to a Java `finally` clause, is `\top`. The `canHandle` predicate is used in another predicate, `isFirstHandler`, which formalises whether or not the given handler (specified by a list of handlers, `\eta`, and an index, `i`), is the correct handler according to indices supplied by the compiler:

\[
isFirstHandler(\eta, i, pc, cl) \equiv \text{canHandle}(\eta(i), pc, cl) \land (\forall j : \text{canHandle}(\eta(j), pc, cl) \Rightarrow j \leq i)
\]

The indices represent an ordering of the handlers that is used to determine which of two (or more) competing handlers should be used when an exception is thrown in their common region of responsibility. The highest index corresponds to the most specific handler.

The function `findHandler` uses `isFirstHandler` to find the address (in this case only the program counter value because the method is given) of the first handler or return `\bot` if no handler is available:

\[
\text{findHandler}(m, pc, cl) = \begin{cases} 
\eta(i).\text{handlerAddr} & \text{if } \eta = m.\text{handlers} \land \text{dom}(\eta) \neq \emptyset \land \exists i : \text{isFirstHandler}(\eta, i, pc, cl) \\
\bot & \text{otherwise}
\end{cases}
\]

4. Control flow analysis

In the following we give an overview of the control flow analysis we have designed for Dalvik. Control flow analysis is an essential component for building more advanced and specialised analyses, e.g., access control and information-flow analyses. But control flow analysis is also useful on its own, e.g., to find methods that are never called. The control flow analysis also has to take advanced control flow concepts such as dynamic dispatch, exceptions and reflection into account since these are integral to Dalvik. The former two are briefly described below while the latter is dealt with in detail in Section 5.

Our control flow analysis is specified as a flow logic [20]. In this approach an analysis is defined through a number of flow logic judgements that specify what is required of an analysis result in order to be sound. Many other frameworks for program analysis exist, but the combination of structural operational semantics and flow logic has proven to be flexible and easy to use for both theoretical developments as well as for implementation. In particular, a similar approach has been used to specify, reason about, and implement numerous analyses for the Java bytecode language [11] which is similar in nature to Dalvik. A more detailed comparison with other approaches is outside the scope of this paper.
4.1. Abstract domains

Before we can define the flow logic judgements for the analysis itself, we first need to define the abstract domains over which the analysis works. The abstract domains are used in the analysis to represent abstractions of runtime values and they closely follow the structure of the underlying semantic domains.

We start by defining the most basic abstract domain, namely that for values. An abstract value, similar to a concrete (semantic) value, can be either an abstract primitive value or an abstract reference:

$$\hat{\text{Val}} = \hat{\text{Prim}} + \hat{\text{Ref}} \quad \text{and} \quad \text{Val} = \mathcal{P}(\hat{\text{Val}})$$

For our analysis, it is sufficient to model abstract primitive values simply as integers, $$\hat{\text{Prim}} = \text{Prim} = \mathbb{Z}$$. Note that we use overlined domain names to indicate an abstract domain based on a concrete domain with the same name while domains with a “hat” (like $$\hat{\text{Val}}$$) indicate domains that are complete lattices. The latter property is important for ensuring that the flow logic specification is well-defined.

Dalvik has native support for string constants that we model simply as $$\hat{\text{String}} = \mathcal{P}\text{(String)}$$ where we assume the existence of a primitive string domain String. The native strings are different from the character arrays that live in java/lang/String objects in the Java standard library and therefore a Dalvik native string cannot, as such, be used in place of a “proper” Java string. However, for the analysis we may consider the (abstract) native strings to be contained in the (abstract) Java string objects. An example of this is shown in the analysis of the const-class instruction in Section 4.2.

Abstract references are either object references, array references, or a null reference:

$$\hat{\text{Ref}} = \hat{\text{ObjRef}} + \hat{\text{ArrRef}} + \{\text{null}\}$$

For object references we take an approach similar to the textual object graphs in [31]: we model all runtime object references as the class of the runtime object combined with the syntactic creation point of the object. Array references are formalised simply as the type of the array, i.e., the analysis merges all information about arrays of the same type:

$$\hat{\text{ObjRef}} = \text{Class} \times \text{Method} \times \text{PC} \quad \text{and} \quad \hat{\text{ArrRef}} = \text{ArrayType}$$

Note that here we directly use the concrete program structure defined in Section 3 to model the abstract program structure. To enhance readability, specific instances of object and array references are written ‘(ObjRef cl, m, pc)’ and ‘(ArrRef t)’ respectively. In anticipation of later developments, we introduce the following notation for extracting all (abstract) references to a particular class:

$$X|_{\text{id}} = \{(\text{ObjRef } (cl’, m, pc)) \in X | cl = cl’\}$$

In order to achieve sufficient precision, the analysis tracks the value of individual registers at every program point, including any return values produced by a method. This leads to the following abstract domains for program points and registers:

$$\hat{\text{PC}} = \text{Method} \times (\text{PC} + \text{END}) \quad \text{and} \quad \hat{\text{Register}} = \text{Register} \cup \{\text{retval, END}\}$$

The token ‘END’ is introduced in the analysis as both a special program counter value and as a special register such that the set of values returned by a method $$m$$ can be easily referenced.

The domain for tracking register values can then be formalised as a mapping for every (abstract) program point from registers to abstract values:

$$\hat{\text{LocalReg}} = \hat{\text{PC}} \rightarrow \hat{\text{Register}} \rightarrow \hat{\text{Val}}$$

For $$\hat{R} \in \hat{\text{LocalReg}}$$ we shall write $$\hat{R}(m, \text{END})$$ as shorthand for $$\hat{R}(m, \text{END})(\text{END})$$, i.e., the (abstract) return value produced by method $$m$$.

Note that tracking the values of local registers at each program point leads to a control flow analysis that is intraprocedurally flow sensitive, i.e., flow sensitive within methods. As already mentioned, this is necessary to achieve sufficient precision for the analysis to be useful. However, it also enables analyses that are intra-procedurally state based such as tracking (within a method) which classes have been initialised.

The subset ordering on $$\hat{\text{Val}}$$ can be extended point-wise to an ordering on $$\hat{\text{LocalReg}}$$. Let $$\hat{R}_1, \hat{R}_2 \in \hat{\text{LocalReg}}, a_1, a_2 \in \hat{\text{PC}}$$, and define

$$\hat{R}_1 \sqsubseteq \hat{R}_2 \iff \forall a \in \text{dom}(\hat{R}_1) : \hat{R}_1(a) \subseteq \hat{R}_2(a)$$

$$\hat{R}_1(a_1) \sqsubseteq \hat{R}_2(a_2) \iff \forall r \in \text{dom}(\hat{R}_1(a_1)) : \hat{R}_1(a_1)(r) \subseteq \hat{R}_2(a_2)(r)$$

In anticipation of the flow logic specification, we define the following notation:

$$\hat{R}_1(a_1) \sqsubseteq_X \hat{R}_2(a_2) \iff \forall r \in \text{dom}(\hat{R}_1(a_1)) : \hat{R}_1(a_1)(r) \subseteq \hat{R}_2(a_2)(r)$$
The semantic heap is modelled using two abstract domains: one that tracks the static fields of objects, called the static heap, and one that tracks references to objects and arrays:

\[
\text{StaticHeap} = \text{Field} \rightarrow \text{Val} \quad \text{and} \quad \text{Heap} = \text{Ref} \rightarrow (\text{Object} + \text{Array})
\]

The abstract domains for objects and arrays can now be defined:

\[
\text{Object} = \text{Field} \rightarrow \text{Val} \quad \text{and} \quad \text{Array} = \text{Val}
\]

We could model the structure of arrays but due to imprecise indexes it is of little benefit to the overall precision of the analysis.

The last abstract domain needed for our analysis is needed to track exceptions that are not handled locally within a method:

\[
\text{ExcCache} = \text{Method} \rightarrow \mathcal{P}(\text{ObjRef})
\]

Finally, we are now able to define the abstract domain for the overall analysis:

\[
\text{CFA} = \text{StaticHeap} \times \text{Heap} \times \text{LocalReg} \times \text{ExcCache}
\]

In the following we specify what it means for an element of the above domain to be a sound analysis of a given program.

### 4.2. Flow logic specification

With all the relevant abstract domains defined, we can now specify the flow logic judgements that define our control flow analysis. Flow logic is a specification-oriented approach to program analysis. Analyses are specified by defining, for every instruction, a *judgement* that formalises when an analysis result is a sound approximation of (the effects of) that instruction [20].

In the remainder of this section, we illustrate the analysis by discussing the judgements for a few interesting instructions and refer to [13] for the full specification. We start with the judgement for the *move* instruction which is one of the simplest. Recall that after a *move* instruction is executed, the destination register \(v_2\) contains the value from the source register \(v_1\) while all others are unchanged. This results in the following judgement:

\[
\begin{align*}
(\hat{S}, \hat{H}, \hat{R}, \hat{E}) \models (m, pc) : \text{move} & \; v_1 \; v_2 \\
\text{iff} & \; \hat{R}(m, pc)(v_2) \subseteq \hat{R}(m, pc + 1)(v_1) \\
& \; \hat{R}(m, pc) \subseteq [v_1] \hat{R}(m, pc + 1)
\end{align*}
\]

To improve readability we adopt the convention that conditions in judgements are implicitly joined by conjunction, i.e., they must all hold in order for the judgement to hold. As a further convenience, we use indentation to indicate the scope of logical variables bound by a quantifier (used in later judgements).

In words, the above judgement states that \((\hat{S}, \hat{H}, \hat{R}, \hat{E}) \in \text{CFA}\) is a sound analysis result for the effects of the *move* instruction located in method \(m\) at address \(pc\) if and only if the following two conditions are fulfilled. The first condition \((\hat{R}(m, pc)(v_2) \subseteq \hat{R}(m, pc + 1)(v_1))\) formalises that the set of abstract values in the source register \(v_2\), at the current address \((m, pc)\), must also be present in the set of abstract values for the destination register \(v_1\) at the following address \((m, pc + 1)\). The latter condition \((\hat{R}(m, pc) \subseteq [v_1] \hat{R}(m, pc + 1))\) requires that all registers, except the destination register \(v_1\) which was updated explicitly, contains at least the same abstract values at the next address as at the current address.

The judgement for the *invoke-virtual* instruction is somewhat more involved: for each possible object the method can be called on (these can be found in register \(v_1\) containing the “this” pointer), the method is resolved (by dynamic dispatch using the *resolveMethod* function from the semantics), the arguments are transferred, and the *retval* register is updated with the return value (unless the return type of the method is void). In addition, the judgement must also deal with exceptions: both the null pointer exception that the instruction may throw itself as well as those that are thrown in but not handled by the invoked method. The latter are tracked using the exception component of the analysis \(\hat{E}\). Exceptions and exception handling is discussed in more detail below:

\[
\begin{align*}
(\hat{S}, \hat{H}, \hat{R}, \hat{E}) \models (m, pc) : \text{invoke-virtual} & \; v_1 \ldots v_n \; \text{meth} \\
\text{iff} & \; \forall (\text{ObjRef}(c, m_i, pc_i)) \in \hat{R}(m, pc)(v_1) : \\
& \; m' = \text{resolveMethod}(\text{meth}, cl) \\
& \; \forall 1 \leq i \leq n : \hat{R}(m, pc)(v_i) \subseteq \hat{R}(m', 0)(m'.\text{numLocals} - 1 + i) \\
& \; m'.\text{returnType} \neq \text{void} \Rightarrow \hat{R}(m', \text{END}) \subseteq \hat{R}(m, pc + 1)(\text{retval}) \\
& \; \forall \text{oref} \in \hat{E}(m') \text{: } \text{HANDLE}_{(\hat{R}, \hat{E})}(\text{oref}, (m, pc)) \\
& \; \hat{R}(m, pc) \subseteq \hat{R}(m, pc + 1) \\
& \; \text{HANDLE}_{(\hat{R}, \hat{E})}(\text{ObjRef}(\text{NullPointerException}, m, pc), (m, pc))
\end{align*}
\]

Two things can happen when an exception is thrown: if a local handler exists, control is transferred to that handler with a reference to the exception object in the *retval* register. If no local handler exists, the method aborts and the exception is
put on the call stack in an exception frame. The following auxiliary predicate, also used in the above judgement, formalises the exception handling semantics:

\[
\begin{align*}
\text{HANDLE}^\beta((\text{ObjRef}(c, e, pc)), (m, pc)) & \equiv \\
\text{findHandler}(m, pc, cl_e) & = pc' \neq \bot \Rightarrow \\
\{\text{ObjRef}(c, e, pc_e)\} & \subseteq \hat{R}(m, pc')(\text{retval}) \\
\hat{R}(m, pc) & \sqsubseteq_{\text{retval}} \hat{R}(m, pc') \\
\text{findHandler}(m, pc, cl_e) & = \bot \Rightarrow \\
\{\text{ObjRef}(c, e, pc_e)\} & \subseteq \hat{E}(m)
\end{align*}
\]

With the above predicate, it is trivial to define the analysis for the \texttt{throw} instruction:

\[
\begin{align*}
\hat{S}, \hat{H}, \hat{R}, \hat{E} & \models (m, pc) : \texttt{throw } v \\
\text{iff } & \forall oref_e \in \hat{R}(m, pc)(v) : \text{HANDLE}^\beta((oref_e, (m, pc)) \\
& \exists \text{ObjRef}(\text{NullPointer}\text{Exception}, (m, pc)).
\end{align*}
\]

As the final example we specify the judgement for the \texttt{const-class} instruction, illustrating both how to handle specialised Dalvik instructions as well as how fields in runtime objects are accessed. Also note how shorthands are introduced for the long names of the Java API:

\[
\begin{align*}
\hat{S}, \hat{H}, \hat{R}, \hat{E} & \models (m, pc) : \texttt{const-class } v \ cl \\
\text{iff } & \forall oref_cl = \text{ObjRef}(\text{java/lang/Class}, m, pc) \\
& \{\text{cl.name}\} \subseteq \hat{H}(oref_cl)(\text{value}) \\
& \{oref_cl\} \subseteq \hat{H}(oref_cl)(\text{name}) \\
& \{oref_cl\} \subseteq \hat{R}(m, pc+1)(v) \\
& \text{dom}(\hat{H})|_{\text{java/lang/Class}} \subseteq \hat{R}(m, pc+1)(v) \\
& \hat{R}(m, pc) \sqsubseteq_{\text{val}} \hat{R}(m, pc+1)
\end{align*}
\]

Following the semantics of the instruction (discussed in Section 3) the class name given as argument \texttt{cl} is stored in the \texttt{value} field of a newly created \texttt{String} object, the reference to which is stored in the \texttt{name} field of a \texttt{Class} object, which is stored in the destination register \texttt{v}. Finally, to take possible object sharing into account, we copy all abstract references to \texttt{java/lang/Class} found in the (domain of) the abstract heap to the result register \texttt{v}.

### 4.3. Correctness of the analysis

We define and prove the semantic soundness of the analysis by means of a subject reduction result. To this end the relationship between the concrete and abstract domains is formalised. We use representation functions to map semantic domains to their abstract domains while keeping the best possible representation. For semantic domains that do not have corresponding abstract counterparts, correctness relations are introduced. We treat the analysis without exceptions. While this simplifies the presentation, the proof extends in a straightforward way to the full semantics.

#### 4.3.1. Representation functions and correctness relations

As values can be either primitive values or references (see Section 4.1) representation of a concrete value is delegated depending on type:

\[
\beta^H_{\text{Val}}(v) = \begin{cases} 
\beta^\text{Prim}(v) & \text{if } v \in \text{Prim} \\
\beta^H_{\text{Ref}}(v) & \text{if } v \in \text{Ref} \\
\bot_{\text{Val}} & \text{if } v = \bot_{\text{Val}}
\end{cases}
\]

Since the representation of a reference value depends on the heap as well as the value, as we shall soon see, the representation function for values in general is parameterised by it as well.

Primitive values are represented as integers in both the concrete and abstract domains (\(\hat{\text{Prim}} = \mathcal{P}(\text{Prim}) = \mathcal{P}(\text{Prim}) = \mathcal{P}(\mathbb{Z})\)) so the best representation is to inject a value into its singleton set: \(\beta^\text{Prim}(p) = \{p\}\).

Since concrete references by themselves are not distinguished by whether they reference an object or an array, the concrete heap is necessary as a parameter to their representation function:

\[
\beta^H_{\text{Ref}}(loc) = \begin{cases} 
\{\text{ObjRef}(cl, m, pc)\} & \text{if } H(loc) \in \text{Object} \land H(loc).class = cl \land H(loc).origin = (m, pc) \\
\{\text{ArrRef} t\} & \text{if } H(loc) \in \text{Array} \land H(loc).type = t \\
\{\text{null}\} & \text{if } loc = \text{null}
\end{cases}
\]
Register contents are modelled as a mapping from registers to values (see Section 3.2), so a register valuation \( R \in \text{LocalReg} \) can be represented by representing the values that the registers map to. Similarly for a static heap \( S \in \text{StaticHeap} \):

\[
\beta^H_{\text{LocalReg}}(R) = \beta^H_{\text{Val}} \circ R \quad \text{and} \quad \beta^H_{\text{StaticHeap}}(S) = \beta^H_{\text{Val}} \circ S
\]

The dynamic heap is more intricate. Since the analysis abstracts object references into classes, an abstract object reference may correspond to more than one concrete location. To handle this the representation function for heaps is also parameterised on an abstract reference, and the representations of the corresponding concrete objects are joined:

\[
\beta^H_{\text{Heap}}(H)(\text{ObjRef}(cl, m, pc)) = \bigcup_{\text{loc} \in \text{dom}(H)} \beta^H_{\text{Object}}(H(\text{loc}))
\]

where the representation function for objects is defined as follows:

\[
\beta^H_{\text{Object}}(o) = \beta^H_{\text{Val}} \circ (o, \text{field})
\]

The representation function for array references is defined in the same way.

**Correctness relations** A stack frame (of domain \( \text{Frame} \)) is correctly represented if the abstract representation of its register valuation is contained in the corresponding abstract register valuation at the right address:

\[
\langle m, pc, R \rangle \; \mathcal{R}^H_{\text{Frame}} \; \hat{R} \; \text{iff} \; \beta^H_{\text{LocalReg}}(R) \subseteq \hat{R}(m, pc)
\]

A call stack is a sequence of stack frames, \( SF = F_1 :: \cdots :: F_n \), that is correctly represented if every stack frame of the call stack is correctly represented:

\[
SF \; \mathcal{R}^H_{\text{CallStack}} \; \hat{R} \; \text{iff} \; \forall 1 \leq i \leq n : F_i \; \mathcal{R}^H_{\text{Frame}} \; \hat{R}
\]

We can now state that a complete configuration is correct if the abstract representations of the heaps are contained in their respective corresponding abstract domains:

\[
\langle S, H, SF \rangle \; \mathcal{R}^H_{\text{Conf}} \; \tilde{\hat{S}}, \tilde{\hat{H}}, \tilde{\hat{R}} \; \text{iff} \; \beta^H_{\text{StaticHeap}}(S) \subseteq \tilde{\hat{S}} \wedge \beta^H_{\text{Heap}}(H) \subseteq \tilde{\hat{H}} \wedge SF \; \mathcal{R}^H_{\text{CallStack}} \; \tilde{\hat{R}}
\]

### 4.3.2. Subject Reduction Theorem

The theorem states that an analysis result that is correct with respect to a semantic configuration remains correct under semantic reduction.

**Theorem 1** (Subject Reduction). Let \( A \in \text{App} \) and \( C \in \text{Configuration} \) such that \((\tilde{\hat{S}}, \tilde{\hat{H}}, \tilde{\hat{R}}) \in \text{CFA}, (\tilde{\hat{S}}, \tilde{\hat{H}}, \tilde{\hat{R}}) \models A, \text{ and } A \vdash C \implies C'\). Then \( C \; \mathcal{R}^H_{\text{Conf}} \; (\tilde{\hat{S}}, \tilde{\hat{H}}, \tilde{\hat{R}}) \implies C' \; \mathcal{R}^H_{\text{Conf}} \; (\tilde{\hat{S}}, \tilde{\hat{H}}, \tilde{\hat{R}})\).

The proof is a tedious but straightforward case analysis on the semantic reductions. We have proved the theorem for a core of the language, see Appendix D. We expect that the proof can be extended easily to the full language. To handle exceptions and the return instruction a condition will have to be added such that the callstack can be trusted to be the result of an actual program execution. A well-formed configuration can be defined as a configuration with such a well-formeded callstack, and it would also have to be shown that well-formedness is preserved under semantic reduction.

### 4.4. Concurrency

There are two Dalvik instructions related to concurrency: monitor-enter and monitor-exit. They are generated by the compiler when the Java keyword `synchronized` is used and they are used in most apps.

In the concurrent execution model, the execution order of instructions is defined by the Java Memory Model, JSR-133 [23], which formalises how shared variables should be read and written, and how instructions can be re-ordered to execute as-if-serially when they are concurrent [19]. According to unofficial statements by one of the Dalvik developers, Alexey Kryshen, on the Stackoverflow website, Dalvik tries to comply with the JSR-133 memory model, though there should be cases where it does not on versions prior to Android 3.0.

In the present work we have formalised neither the semantics nor the analysis of concurrency and the related instructions. However, we conjecture that our control flow analysis can be extended to a sound analysis even for multi-threaded apps since the analysis is flow insensitive with respect to heap access. Therefore the analysis already represents an over-approximation of all possible interleavings of all heap accesses, including those that would otherwise be added by allowing multi-threaded apps.

---

Reflection

Reflection allows a program to access class information at runtime, and use this information to create new objects, invoke methods or otherwise change the control flow of the program. When reflection is used, the types involved are usually not known statically. Instead, they are retrieved dynamically from strings. The strings can come from sources such as user input, files included with the app, the Internet, or, in some cases, constant strings in the program. We found that several of the apps in our data set specify constant strings in the program.

The most used method from the Java reflection API is Method.invoke(). It is an instance method on the java/lang/reflect/Method class used to invoke dynamically resolved methods. An example can be seen in Listing 1 where the method bar(int) on the class pkg.examples.Foo is invoked on an instance of the class with the argument '3'.

```java
1 Class<?> clazz = Class.forName("pkg.examples.Foo");
2 Method method = clazz.getMethod("bar", int.class);
3 Integer result = (Integer) method.invoke(clazz.newInstance(), 3);
```

Listing 1: A method invoked through reflection in Java.

A Method object can be retrieved using the instance method getMethod() on the Java standard class java/lang/Class. The instance of Class does not have to represent a class that implements the method in question since it will be resolved with dynamic dispatch like normal method calls. In Listing 1, the Class object is retrieved using the static method Class.forName() that, given a fully qualified class name, returns a reference to a Class instance for the specified class.

Another way to obtain a Class object is through the Dalvik instruction const-class (see Section 3 for the semantics of this instruction). It is generated when the static field class, which is found in all Java classes, is accessed. An alternative to Class.forName("pkg.examples.Foo") at line 1 in Listing 1 would therefore be Foo.class, given that the example code is located in the same package as the Foo class and that the class can be found at compile time.

The Method objects are mainly retrieved using the methods getMethod() and getMethods(). The latter returns an array of all public method declarations on a class while the former returns a single object that is found by specifying the name and parameter types of the desired method declaration. The methods getMethod() and getMethods() only find public method declarations. They both find the method declaration objects by traversing the class hierarchy, starting at the class represented by the Class object and searching upwards through superclasses and interfaces. We uncovered and reported an undocumented change in the semantics of this traversal which we discuss in Section 5.5. Developers can also use the getDeclaredMethod() and getDeclaredMethods() methods which only look in the given class but may also return private methods.

Once a Method object has been obtained, it can be used to retrieve information about the method declaration, for example access modifiers, name, and the (checked) exceptions it can throw. Accessing these requires no additional information besides that which is known statically from the method declaration. To invoke the method, an instance of the class or a subclass is required (except for static methods). The receiver object can be any Java object created using the regular Java new statement or through the newInstance() method on a Class object. In addition to creating a new instance, the newInstance() method calls the parameterless constructor for the class. To use another constructor, an instance of the Constructor class from the reflection API must be used.

Reflection in the wild

We have run an automated informal search for constant strings supplied to the two java/lang/Class methods forName() and getMethod(). Of the 150 apps in three randomly selected market categories (News and Magazines, Photography, and Productivity),

- 18.7% of the apps use neither forName() nor getMethod() (meaning no method calls are made with variable strings)
- 17.3% use at least one of the methods and use only constant strings
- 25.3% use exactly one variable string in total for both methods
- 38.7% use more than one variable string in total for both methods.

Here we searched for instances of the two Class methods with a const-string instruction in the preceding vicinity, writing to the same register read by the method. This approach gives a number of false positives, most prominently those that simple constant propagation would find. But also in subtler cases, as seen in the 37.3% of the apps containing a class named InstallReceiver. This class uses forName() with a variable string, but only if it equals the constant "com.google.android.apps.analytics.AnalyticsReceiver". So with the techniques we formalise in the following sections, it is possible to analyse more than a third of real-world apps. Handling of (the effects of) simple patterns of reflection usage may increase this number further (as indicated by the large number of apps that only sparingly use variable strings for reflection).
5.2. Usage patterns

The use-cases for reflection vary from app to app, and Android developers use it for many different things. However, we have observed some patterns in usage, most of which we have found through manual inspection of the bytecode.

Hidden API methods are invoked. Certain features in Android are deliberately hidden such that they are not present in the JAR file for the Android API that app developers use when compiling their Android apps. This is typically done when an API is not yet considered stable or if the underlying functionality is not implemented on all devices intended to support it. A prominent example is the Bluetooth API, most of which was hidden in the early releases of Android. Developers tend to use these features anyway, and use reflection to do so instead of precompiling their own fully featured JAR file for the Android API.

Private API methods and fields are accessed by bypassing access modifiers. Several features of the Android platform are placed in private methods and fields, such as the ability to create a list of text messages from raw SMS data.

Backward compatibility. As new versions of Android are often released with new features, developers tend to use reflection to check if certain methods/features exist and call these only when they do. This pattern is even encouraged by Google.

JSON and XML is generated and parsed with the use of reflection. Some apps use JSON and XML that contain information about their Java objects, and through reflection generation and parsing can be automated.

Libraries for Android apps are widely available on the Internet, and some of them use reflection. In many apps that use reflection, it is only used by the included libraries.

In [17] a reflection analysis for standard Java is defined and discussed as well as the use of reflection in a number of large open source projects. This study revealed some of the same patterns as those reported here, such as object serialisation and portability/backward-compatibility. However, it was found that reflection was mostly used to create new objects without invoking new methods on them. In contrast, we found that in Android, invoking methods is among the most common uses of reflection. Our findings are however consistent with those reported for Android in [8].

5.3. Assumptions

Static analysis of the reflection API is not possible in all cases. We therefore make the following assumptions:

- All classes used through reflection are known statically, such that its components can be analysed. In other words, we assume that dynamic class loading is not used.
- The program does not use a non-default class loader, as this could change the behaviour of Class.forName() and related methods. As mentioned in Section 2, class loaders are used in 13.1% of the apps to either load or define new classes. This raised other problems with regards to static analysis, and they were considered out of scope for this project.
- The strings used to obtain Method and Class objects used for reflection can be determined statically. This presumption only holds for some of the studied apps. A preliminary rough analysis of our data set shows that 57% and 44% of all calls to Class.forName() and Class.getMethod() respectively use locally defined constant strings. Of the studied apps, a total of 19% exclusively use locally defined constant strings for all calls to the methods Class.forName() and Class.getMethod()..

These last results are based on a sampling of const-string instructions syntactically appearing shortly before the invocation of the reflection methods. They are not based on the limited but inter-procedural data flow capabilities of our control flow analysis. Even with an inter-procedural analysis, improving the numbers requires the ability to track strings across collection APIs such as java/util/ArrayList and follow string manipulation such as that of the java/lang/StringBuilder class. An example of the former is discussed briefly in Section 8. For the latter, existing string analyses such as [5,15,26] may prove useful.

The operational semantics specified so far all represent single Dalvik instructions. We now change focus and specify operational semantics and flow logic judgements to represent Java API method calls. This has two reasons: (1) Using our analysis to analyse the Dalvik instructions that the API method consists of is not possible for APIs implemented natively in C or C++. (2) Analysing reflection at the level of API calls makes it simpler and more precise to recognise the special patterns employed when using reflection. Therefore, we have chosen to specify the operational semantics for the API methods as if

---


20 When our original work was done, the description of reflection for backward compatibility was part of the official Android developer’s website at http://developer.android.com/resources/articles/backward-compatibility.html but the page has since been removed. The same content is now available at the Android Developers Blog, controlled by the Open Handset Alliance, at http://android-developers.blogspot.com/2009/04/backward-compatibility-for-android.html.
they each were single, although advanced, Dalvik instructions. Most of the reflection API calls can throw exceptions, but for presentation purposes we only describe these in some cases.

5.4. Class objects

When the Java method `Class.forName(string)` is used, it generates the Dalvik instruction `invoke-static` with the signature

```
Ljava/lang/Class; -> forName(Ljava/lang/String;)* -> Ljava/lang/Class;
```

For readability in the semantics and judgements, we identify such specialised calls using `meth = java/lang/Class->forName`, with the method signature shortened. For this method the `invoke-static` instruction takes one register argument: a reference to a string that identifies the class or interface one wants to refer to as a `Class` object. As is the case with the `const-class` instruction (see Section 3.3), the corresponding class object may already exist. For this rule we additionally use the auxiliary function `lookupClass` to find the class in the semantic Class domain from the supplied fully qualified class name. If no class object exists a new one is allocated on the heap and the `name` field is updated to point to the string reference of the class name:

```
m.instructionAt(pc) = invoke-static v1 meth
c1 = lookupClass(o.field(value)) ∈ Class
H', loccl = findClassObject(H, c1) = ⊥
o'_d = o_d[field → o_d.field[name → loc]]
R' = [retval → loccl]
```

Since no new frame is pushed on the stack, the program counter is incremented directly. If the class object already exists, a reference to it is returned:

```
m.instructionAt(pc) = invoke-static v1 meth
c1 = lookupClass(o.field(value)) ∈ Class
H', loccl = findClassObject(H, c1) = loccl ≠ ⊥
```

For the analysis, register `v1` may contain several values but we can safely ignore anything other than strings, as the API method will only accept a string as an argument. Every string reference in `v1` is transferred to a new location on the heap, into the field `name` on the object identified by the type `java/lang/Class`, the current method and program counter. To take object sharing into account we copy all references to `java/lang/Class` (including the newly created one) to the `retval` register:

```
(Š, Š', Š, Š') ⊨ (m, pc): invoke-static v1 meth
iff meth = java/lang/Class->forName
oref = ObjRef(java/lang/Class, m, pc)
∀oref ∈ Š(oref) ∈ `java/lang/String`: [oref] ⊆ Š(oref)(name)
dom(Š)|java/lang/Class ⊆ Š(m, pc + 1)(retval)
Š(m, pc) ∈ [retval] Š(m, pc + 1)
```

5.5. Method objects

A `Method` object represents a method declaration, i.e., an element in the `MethodDeclaration` domain. This means that `Class.getMethod()` finds a method declaration resolved from the class or interface represented by the `Class` object. Two auxiliary functions are used in the semantics to handle method resolution and object creation respectively: `resolvePublicMethodDeclaration` and `newMethodObject`.

The `resolvePublicMethodDeclaration` function takes as arguments a class name, a method name, and a type signature (in total, a method signature), and searches through the class and interface hierarchy for a matching method declaration. The function only finds methods that are defined as public and which are not constructors.

The function `newMethodObject` is given the method declaration and the existing heap and returns the updated heap and the location of the `Method` object where the relevant fields have been initialised. In fact it creates three new objects:
a Method, a Class, and a String, because the field declaringClass on the Method object references a Class where the field name references a String with the actual class or interface name:

\[
m.\text{instructionAt}(pc) = \text{invoke-virtual } v_1 v_2 v_3 \text{ meth}
\]

\[
\text{meth} = \text{java/lang/Class->getMethod} \\
\text{clname}_0 = H(R(v_1)).\text{field}(\text{name}) \quad \text{clname} = H(\text{clname}_n).\text{field}(\text{value}) \\
mname = H(R(v_2)).\text{field}(\text{value}) \quad \text{types} = H(R(v_1)).\text{field}(\text{value}) \\
m = \text{resolvePublicMethodDeclaration}(\text{clname}, \text{mname}, \text{types})
\]

\[
m \neq \bot \quad (H', loc_m) = \text{newMethodObject}(H, m)
\]

\[
A \vdash (S, H, (m, pc, R) \vdash SF) \implies (S, H', (m, pc + 1, R[\text{retval} \mapsto \text{loc}_m]) \vdash SF)
\]

We have defined the search order of the resolvePublicMethodDeclaration function as follows: The current class, the superclasses of the current class and finally the interface hierarchy of the current class. The interface hierarchies of the superclasses are not searched despite the fact that methods declared in these would be found if standard method resolution was used instead of reflection. This unexpected behaviour is consistent with the Java documentation [22] and the behaviour in Android 2.3. However, the search order has changed in Android 4.0 to be consistent with the “natural” behaviour where interfaces of superclasses are searched as well. This change in behaviour is undocumented and we have reported this as a bug.\(^{21}\) Regardless of search order, the function is able to find more than one applicable method declaration due to covariant return types. In such cases, the one with the most specific return type is returned, and if a single return type is not more specific than the others, an arbitrary method declaration is returned.

In the analysis, we define mref for readability to be the reference to the new Method object. For all references to Class objects in \(v_1\), there are one or more class names referenced by a String on the heap in the field name. The set of class names is bound to clnames. For each of the String references from \(v_2\) the string values (method names) are bound to mnames. The set of method names is also put in the field name on the heap at mref. Furthermore, we use resolvePublicMethodDeclarationsFromNames to do a search through the class and interface hierarchy for valid method declarations, similar to the semantic resolvePublicMethodDeclaration, but for sets of class and method names. However, it does not take argument types into account since we do not model arrays precisely enough to do a reasonable comparison of the argument types. For each of the resulting method declarations \(\{m'\}\): the class name of the method declaration is created as a String, the string reference is put into a (newly created) Class object. Taking potential object sharing into account, all references to the Class object are copied to the field declaringClass for mref on the heap. Finally, a reference to the method object is present in the retval register:

\[
(S, H, E) \models (m, pc) : \text{invoke-virtual } v_1 v_2 v_3 \text{ meth} \\
\text{iff } \text{meth} = \text{java/lang/Class->getMethod} \\
\text{mref} = (\text{ObjRef (java/lang/reflect/Method, m, pc)}) \\
\forall \text{mref}' \in \hat{R}(m, pc)(v_1) | \text{java/lang/Class} : \\
\forall \text{mref}'_G \in \hat{H}(\text{mref}'_G)(\text{name}) | \text{java/lang/String} : \\
\text{clnames} = \hat{H}(\text{mref}'_G)(\text{value}) \\
\forall \text{mref}'_G \in \hat{R}(m, pc)(v_2) | \text{java/lang/String} : \\
\text{mnames} = \hat{H}(\text{mref}'_G)(\text{value}) \\
\text{mref}' \in \text{resolvePublicMethodDeclarationsFromNames}(\text{mnames}, \text{clnames}) : \\
\forall \text{mref}' \in \text{mref}'_G \subseteq \hat{H}(\text{mref}')(\text{name}) \\
\text{dom}(\text{H})(\text{java/lang/Class, m, pc})(\text{name}) \\
\hat{H}(\text{ObjRef (java/lang/String, m, pc)}) \subseteq \\
\hat{R}(\text{ObjRef (java/lang/Class, m, pc)})(\text{name}) \\
\{\text{mref}'\} \subseteq \hat{R}(m, pc + 1)(\text{retval}) \\
\hat{R}(m, pc) \subseteq \text{retval} + 1
\]

6.5. Instantiation

The API method Class.newInstance() is used to instantiate new objects through reflection. It requires a Class object representing the class to be instantiated. The class must be a regular class, not an interface, abstract class, primitive type or array class. In such cases, an exception is thrown (this is left out of the semantics and analysis to simplify presentation but could easily be added). The Class object contains a fully qualified class name and we use the auxiliary function lookupClass to find the corresponding class in the semantic Class domain. Next, the new instance is created on the heap using the same function newObject as in the regular new-instance instruction. Unlike the new-instance instruction,

\(^{21}\) See the Android Issue Tracker (issue 31485) at \text{http://code.google.com/p/android/issues/detail?id=31485}.
Class.newInstance() also calls the default constructor for the class being instantiated. We use an auxiliary function lookupDefaultConstructor to find this constructor, and if none exists the function will return ⊥ and an exception should be thrown. The constructor is given registers where the argument register has been initialised to a reference to the newly allocated object. Control is transferred to the constructor by adding a new stack frame, just like regular method invocation, but a reference to the newly allocated object is also put into the retval register on the stack frame for the current method. A constructor cannot return a value, and therefore this reference cannot be replaced before control is returned to the current method:

\[
\begin{align*}
m\text{.instructionAt}(pc) &= \text{invoke-virtual } v_1 \text{ meth} \\
meth &= \text{java/lang/Class->newInstance} \\
loc_c = R(v_1) \neq null \\
o_d = H(loc_c) \\
(H',loc) = \text{newObject}(H,cl,m,pc) \\
m' = \text{lookupDefaultConstructor}(cl) \neq ⊥ \\
R' &= [0 \mapsto ⊥, \ldots, m'.numLocals - 1 \mapsto ⊥, m'.numLocals \mapsto H'(loc)]
\end{align*}
\]

The flow logic judgement specifies that for all the Class references given to the method, String references exist on the heap to specify the class name, and for each of these class names (clname) the semantic class must be found using the function lookupClass. A reference for each of these classes is put into the retval register for the current method, a default constructor is found and the new object reference is placed as an argument to the constructor:

\[
\begin{align*}
\{S,H,R,\hat{E}\} \models (m,pc)\!:\text{invoke-virtual } v_1 \text{ meth} \\
\text{iff meth} &= \text{java/lang/Class->newInstance} \\
\forall o_{ref} \in \hat{R}(m,pc)(v_1)\!|\!\text{java/lang/Class} : \\
\forall o_{ref'} \in \hat{H}(o_{ref})(\text{name})\!|\!\text{java/lang/String} : \\
\forall clname \in \hat{H}(o_{ref'})(\text{value}) : \\
cl &= \text{lookupClass}(clname) \\
\{\text{ObjRef}(cl,m,pc)\} \subseteq \hat{R}(m,pc+1)(\text{retval}) \\
m' &= \text{lookupDefaultConstructor}(cl) \\
\{\text{ObjRef}(cl,m,pc)\} \subseteq \hat{R}(m',0)(m'.numLocals) \\
\hat{R}(m,pc) \subseteq_{\text{retval}} \hat{R}(m,pc+1)
\end{align*}
\]

5.7. Method invocation

Once a Method object is created it can be used to invoke the method it represents. The API method Method.invoke() takes two arguments besides the Method object (v_1) itself: an object reference (v_2) for the receiver object on which the method should be invoked, and an array of arguments (v_3). The receiver object should be null if the method is static, and the method implementation will then be resolved from the declaring class in the Method object. We do not formalise the invocation on static methods as this is a straightforward simplification of the case with a receiver object. We use the auxiliary function methodSignature to extract information from a Method object to create a corresponding signature in the semantic MethodSignature domain. The actual method to invoke is resolved using resolveMethod, just like in the regular invoke-virtual instruction:

\[
\begin{align*}
m\text{.instructionAt}(pc) &= \text{invoke-virtual } v_1 \text{ v2 v3 meth} \\
meth &= \text{java/lang/reflect/Method->invoke} \\
R(v_1) &= loc_c \neq null \\
loc_1 = H(loc_1) \\
loc_2 = H(loc_2) \\
meth' &= \text{methodSignature}(H,loc_1) \\
R(v_2) &= loc_2 \neq null \\
loc_2 = H(loc_2) \\
R(v_3) &= loc_3 \\
loc = H(loc_c) \subseteq \text{Array} \\
m' &= \text{resolveMethod}(meth',loc.class) \\
a' &= \text{unboxArgs}(a,m'.argTypes,H) \\
bf &= \text{boxingFrame}(m'.returnType) \\
R' &= [0 \mapsto ⊥, \ldots, m'.numLocals - 1 \mapsto ⊥, m'.numLocals \mapsto a'.value(0), \ldots, \\
m'.numLocals + a'.length - 1 \mapsto a'.value(a'.length - 1)]
\end{align*}
\]

Before the arguments are transferred to the resolved method registers they may have to be unboxed: the API method (Method.invoke()) receives the arguments in an array with elements of type Object (Java varargs). This means that if the invoked method has any formal arguments of primitive types, the API method unboxes the primitive values that were boxed before the call occurred. The primitive values are extracted from the box object based on the argument types of the resolved method. We use an auxiliary function, unboxArgs, to unbox all the relevant arguments and return an array with the correctly typed values. These values are then transferred into the relevant registers that are put into a new stack frame along with the method to invoke. The unboxed array, a', is longer than a if any of the unboxed values are of wide data types, i.e., long or double.
The API method always returns a value of type Object, and if the invoked method returns a primitive value it must therefore be boxed by the API method. The return value is not available until the invoked method returns, and therefore we cannot yet box the value. Instead, we add an additional stack frame with a special method to be run after the invoked method. We use an auxiliary function boxingFrame to generate this frame. The function takes the return type of the invoked method as an argument, such that the boxing method is able to determine if the return value should be boxed, and what class it should be boxed in. If boxing is to occur, it boxes the return value from the retval register and replaces it with a reference to the boxed value. The boxing method then returns as a regular method by incrementing the program counter of the frame below and updating its retval register.

In the analysis, for all the Method object references in v1, we use the auxiliary function methodSignatures to extract and create all possible method signatures that correspond with the information on the heap for the given Method object. All these method signatures must be resolved on all the object references for receiver objects in v2. We do not store the order of the arguments in the array referenced in v3, and therefore we cannot determine which of the arguments that must be unboxed. Instead, we transfer all values as they were, as well as unboxing all arguments that are object references, if the class (cl0) is a class that can be unboxed. The latter is determined by the auxiliary function isBoxClass. Depending on the return type of the invoked method, the return value of Method.invoke() is either null (if void), passed unchanged (if it is already of a reference type) or boxed (if it is of a primitive type). The function primToBoxClass translates a return type to the corresponding boxing class, e.g. int to Integer, and the return value of the method is then boxed by putting the value in the field value on the heap for the found class and the current method and program counter. In addition, the same object reference is put in the retval register for the next program counter in the current method. Finally, we handle any exceptions that are referenced in the exception cache since the invoked method might have thrown an exception:

\[
(\hat{S}, \hat{H}, \hat{R}, \hat{E}) \models (m, pc) : \text{invoke-virtual } v_1 v_2 v_3 \text{ meth}
\]

iff \( m \text{h} = \text{java/lang/reflect/Method} \rightarrow \text{invoke} \)

\( \forall \text{aref}_{\text{m}} \in \hat{R}(m, pc)(v_1) | \text{java/lang/reflect/Method} : \)

\( \forall \text{meth}' \in \text{methodSignatures}(\hat{H}, \text{aref}_{\text{m}}) : \)

\( \forall \text{ObjRef}(cl, m, pc) \in \hat{R}(m, pc)(v_2) : \)

\( m' = \text{resolveMethod}(\text{meth}', cl) \)

\( \{\text{ObjRef}(cl, m, pc)\} \subseteq \hat{R}(m', 0)(m'.\text{numLocals}) \)

\( \forall 1 \leq i \leq \text{arity(meth')}: \)

\( \forall \text{aref} \in \hat{R}(m, pc)(v_3) \cap \text{ArrRef} : \)

\( \hat{H}(\text{aref}) \subseteq \hat{R}(m', 0)(m'.\text{numLocals} + i) \)

\( \forall \text{ObjRef}(cl, m, pc) \in \hat{H}(\text{aref}) : \text{isBoxClass}(cl) \Rightarrow \)

\( \hat{H}(\text{ObjRef}(cl, m, pc))(\text{value}) \subseteq \hat{R}(m', 0)(m'.\text{numLocals} + i) \)

\( m'.\text{returnType} = \text{void} \Rightarrow \beta(\text{null}) \subseteq \hat{R}(m, pc + 1)(\text{retval}) \)

\( m'.\text{returnType} \in \text{RefType} \Rightarrow \exists \hat{R}(m', \text{END}) \subseteq \hat{R}(m, pc + 1)(\text{retval}) \)

\( m'.\text{returnType} \in \text{PrimType} \Rightarrow \)

\( cl_0 = \text{primToBoxClass}(m'.\text{returnType}) \)

\( \hat{R}(m', \text{END}) \subseteq \hat{H}(\text{ObjRef}(cl, m, pc))(\text{value}) \)

\( \{\text{ObjRef}(cl, m, pc)\} \subseteq \hat{R}(m, pc + 1)(\text{retval}) \)

\( \forall \text{aref}_e \in \hat{E}(m') : \text{HANDLE}_{\hat{R}, \hat{E}}(\text{aref}_e, (m, pc)) \)

\( \hat{R}(m, pc) \subseteq \text{\{retval\}} \hat{R}(m, pc + 1) \)

6. Prototype implementation

The control flow analysis, specified as flow logic judgements, is not in itself immediately useful for analysing actual Dalvik bytecode apps. In this section we describe a prototype implementation of a tool that converts the flow logic judgements into a Prolog program that can then be executed in order to compute an analysis result.

Our prototype combines several existing tools with our Python-based parser and constraint generator as shown in Fig. 1. First, apktool extracts the bytecode content of an app and, leveraging another tool, baksmali, translates the bytecode to smali, a human readable format akin to assembly languages with instruction mnemonics, inlined constants and various annotations. We feed this output to our parser which builds lists of classes, methods, instructions, etc., and a tree representing the type hierarchy in the app. Our constraint generator traverses the lists and emits Prolog rules for method resolution, exception handlers, entry points, etc., as well as rules for each instance of each Dalvik instruction in the program. The Prolog program can then be queried for any information that the analysis specifies. This can for example happen interactively or as part of a more specific, programmed analysis. As an example of the latter, we generate call graphs with a special query and further process the output to visualise the call graph of an app.

The source code for the prototype is available at: https://bitbucket.org/erw/dalvik-bytecode-analysis-tool.
6.1. Examples

Here we demonstrate the conversion of flow logic judgements to Prolog source code. For example, the judgement for the \texttt{const} instruction is:

\[
(\hat{S}, \hat{H}, \hat{R}, \hat{E}) \models (m, pc) : \text{const } v \ c \\
\text{iff } \beta(c) \subseteq \hat{R}(m, pc+1)(v) \\
\hat{R}(m, pc) \sqsubseteq_{[v]} \hat{R}(m, pc+1)
\]

For a \texttt{const} instruction located at PC 48 in method \texttt{m1} in some app, these two Prolog clauses will be generated:

```prolog
\% 48: const v5, 0x1
1 hatR(m1, 49, 5, 0x1).
2 hatR(m1, 49, V, Y) :-
3 hatR(m1, 48, V, Y),
4 hatR(m1, 48, V, Y),
5 V \mapsto 5.
```
As can be seen, this conversion and instantiation is fairly straightforward.

An example of a slightly more advanced instruction is `iput` which sets an instance field to a given value on a given object, provided the object’s class matches the one that is part of the (fully qualified) field name. Its flow logic judgement is:

\[
(\hat{S}, \hat{H}, \hat{R}, \hat{E}) \models (m, pc): iput \ v_1 \ v_2 \ fld
\]

iff \( \forall (ObjRef (cl, m', pc')) \in \hat{R}(m, pc)(v_2): cl \leq fld.class \Rightarrow \hat{R}(m, pc)(v_1) \subseteq \hat{H}(ObjRef (cl, m', pc'))(fld) \)

\[
\hat{R}(m, pc) \subseteq \hat{R}(m, pc + 1)
\]

As an instantiation of the Prolog code for `iput`, we show one from the method `Lru/watabou/moon3d/MoonView;->targetSunAngle:F`

```
1 % 3: iput v1, p0, Lru/watabou/moon3d/MoonView;->targetSunAngle:F
2 hatH((CL, CLAddrMID, CLAddrPC), 'name', 'int').
3 hatR(m2, 3, (CL, CLAddrMID, CLAddrPC)).
4 subclass(CL, 'Lru/watabou/moon3d/MoonView;'),
5 hatR(m2, 3, 1, Y).
6 hatR(m2, 4, V, Y) :-
7 hatR(m2, 3, V, Y).
```

The references from register `p0` at the current program counter are extracted. The subclass condition check is implicit: if the `subclass/2` goal fails, the `hatH/3` relation does not hold for those particular arguments and another reference from register `p0` can be tried. If it succeeds, the variable `Y` is bound to each value from the source register in turn. Also, all registers are transferred to the next program point.

### 6.2. Modelling Java and Android

This section discusses features and program components that are not implemented in the apps and must therefore be modelled separately.

#### 6.2.1. API methods

Methods that are called but not implemented in an app may be from Java standard classes, Android APIs, and from other apps signed by the same developer key. They can also come from classes loaded at runtime but apps that do this are not amenable to static analysis before installation in the first place as discussed in Section 2.

Without handling external methods in some way, it is not possible to resolve the implemented methods. The potential effects of a single method are far-reaching. Using reflection, any method, including private ones, can be called, and anything reachable on the heap from the references given to the unknown method can be changed, again including private and final fields. It is even possible that debugging or diagnostics APIs allow programmatic access to the full heap.

In a specialised analysis it would be useful to be able to trust Java and Android API methods not to do anything malicious, for example by just setting their return values to top, but due to the possibilities of affecting the heap, every API method would require some inspection to determine its effects on the heap. APIs can also be handled as we have done with parts of the reflection API by modelling the methods with individual flow judgements. Alternatively they could be compiled to Dalvik bytecode and analysed along with the app. This last approach might impact the running time of the analysis by increasing the effective size of the app considerably and would only work for the parts of the Java standard classes that are implemented in Java.

#### 6.2.2. Java features

One Java feature we needed to represent is the `java/lang/Class` instances that represent primitive types. They are stored as static fields named `TYPE` on the corresponding box classes, so an `sget` instruction is produced by the Dalvik compiler instead of `const-class`. For example, the `java/lang/Class` instance representing the `int` type is stored as `Integer.TYPE`. We model this with two Prolog facts like the following for each of the eight primitive Java types plus `void`:

```
1 hatH(('Ljava/lang/Class;', 'Lru/watabou/moon3d/MoonView;'), 'name', 'int').
2 hatR((CL, 'java/lang/Class;'), 'name', 'int').
```

Here, `(java, 1)` is introduced as the representation of the `java/lang/Class` instance. The program counter values from 1 to 9 are used for the 9 objects.

---

\(^{22}\) Registers numbered by parameter, such as `p0`, are aliases for other registers. In this method `p0` corresponds to `v2` (because `numLocals` is 2). We perform this translation prior to generating the Prolog code.
6.2.3. Reflection

In order to make analysis of apps with reflection tractable, we rely on the assumptions about the use of reflection made in Section 5.3. Briefly, they are: all classes used through reflection are known statically, the program only uses the standard class loader, and strings used to obtain object representations of methods or classes can be determined statically.

Essentially these assumptions ensure that all uses of reflection can be resolved statically. Apps that break the first and the last assumption cannot easily be analysed, if at all. Apps that use a non-standard class loader may still be analysed, depending on the specifics of the class loader used. Intuitively, if the used class loader implements a restricted subset of the standard class loader it can be handled by the current implementation. For other class loaders, that do not work like the standard class loader, a formalisation and/or representation of that class loader is needed to enable analysis with our prototype.

6.2.4. Entry points

Android apps are simply collections of classes with methods that can be called by the Android system. As an immediate consequence, apps do not have a single uniquely determined entry point. Furthermore, parameters can be passed to the entry points, both from the libraries and the Android system but also from within the app itself. In order for an analysis to soundly capture all of this information, not only the app but the entire system would have to be formalised and analysed. Although an interesting challenge, such comprehensive modelling is out of scope for our analysis and prototype. Instead we opt for an approximate solution in which we simulate calls to potential event handler methods.

We have identified 1695 event handler methods in the Android API such as onCreate() for activities, onLocationChanged() for location services, or onPictureTaken(). Some are declared on interfaces, others on classes. We generate Prolog facts to simulate calls to the methods on all classes in the app that implement one of the relevant API interfaces and on all subclasses of the relevant API classes. We also call the constructors on subclasses of the four main app components that are instantiated by Android: activities, services, broadcast receivers, and content providers. The remaining many minor app components are listeners that are instantiated by the app itself before they are registered such that Android can call them.

All of these entry point methods are instance methods and as a simple over-approximation, we invoke the methods on all object references of the class, the method is implemented in, using the Prolog anonymous variable to ignore the creation point. The arguments passed to the entry point methods are $T_{\text{Prim}}$ for primitive arguments. For arguments of reference type, objects with the artificial creation point (android, 0) are passed to show that the exact creation point of the argument is unknown and could be in the Android system itself.

Class constructors (see Section 3.1) also form entry points but they do not need to be called explicitly as they do not depend on arguments. All instructions generate constraints whether the method they reside in is called or not.

7. Evaluating the prototype

Here we perform an experiment to examine the practical aspects of our formal analysis as implemented in the prototype. Therefore we only take the formalised features into account, hence APIs and libraries are not accounted for. Reflection is handled in the limited ways described in Section 5. While the exception analysis has been formalised it is excluded here as well. Yet, while unsound, our prototype has a stronger theoretical foundation than any tool we have studied so far.

We have tested the prototype with many forms of interactive querying and with call graph extraction on several apps from our data set, and in practice memory usage is a bigger issue than running time. As an evaluation of the performance of the prototype we show in Fig. 2 memory usage during call graph extraction, i.e., the computation of the domain MethodCall = Method $\rightarrow P$(Method). Here, the formalised analysis (CFA as defined in Section 4.1, sans the exception component) is used to find the object references that govern dynamic dispatch as well as values that affect reflective calls.

The app size used in Fig. 2 is measured in the number of methods, and the evaluation is based on apps with up to 2000 methods in the three market categories News and Magazines, Photography, and Productivity.

A memory ceiling of 5 GiB was set and experiments that hit the ceiling and were aborted are indicated. The data shows the expansion by doubling of the evaluation stacks in the XSB Prolog engine with groupings of values around 1, 2, and 4 GiB. The values lying between these groups result from the two main stacks being expanded independently. Analysis of approximately half of the apps with more than 1000 methods exceeds the memory limit.

The CPU time of the experiments that completed within the memory limit ranged from 0.05 to 1227 s with 89% of the runs being below 200 s on an Intel Core i7 processor. The time used for extraction and disassembly of the bytecode from the APK file is negligible and not included. The conversion from smali source to Prolog is included and typically takes less than 10 s for the largest apps in this experiment. Most of the running time is spent compiling the generated Prolog source and evaluating the call graph query, with time distributed almost equally between these two tasks. Consequently, subsequent queries will be answered much faster than the first one for an app.

Turning the above experiment into an estimate of the percentage of real-world apps the prototype can handle, Fig. 3 shows the distribution of app sizes in our data set. Apps with 1000 methods or less comprise 42% of the data set, indicating that the prototype in its current state can analyse this part of the apps with 5 GiB of memory at its disposal. Fig. 2 reveals an exponential tendency, suggesting both that almost the same number of apps can be handled with only 2.5 GiB memory, for example, but also that exceptionally large apps with 20,000–50,000 methods will use disproportionate amounts of memory.
Indeed our tests reveal that some apps require too much memory to analyse, even on a server with 68 GB memory available to us.

Since we have implemented and improved the analysis simultaneously, readability and debuggability of the Prolog code has been important to us. The prototype is strictly proof-of-concept but we expect that it would be possible to improve the efficiency of the analysis considerably.

The large majority of clauses are concerned with copying register values for every step of a method. An optimisation would concern copying values directly to the program points where they are read, thus bringing down the number of clauses significantly. Many Dalvik instructions produce runtime exceptions and their clauses often make up one third of the generated Prolog program. This is another opportunity for more efficient handling. In the above experiments runtime exceptions were not considered.

Currently the solver runs only a single thread so with respect to running time there are big gains to be had by parallelising the computation appropriately, as has been done for the analysis of timed automata in [6,16]. In [21] it is shown...
how the framework can be extended to general program analysis, such as the analysis specified in this paper. The parallel implementation of the framework is shown to scale almost linearly, up to a factor of 40 on a 48-core machine [6].

Another implementation approach, similar in spirit to our current approach, would be to use a Datalog solver. Prolog allowed us to rapidly design and implement a proof-of-concept prototype to validate the analysis as well as the implementation strategy of generating constraints. Using Datalog is a suitable avenue for future work.

8. Looking for malware

As an example of an approach to malware detection, we have examined apps that send text messages to see which phone numbers are used as destinations. In many cases, it was not possible to learn the numbers because they come from API methods that we do not yet support. The typical pattern for legitimate apps that send text messages is to retrieve numbers from the database with the user's contact list. This requires a large number of API calls, some to connect to the database, some to read the content, and some to store and retrieve the results from Java collections. However, to test tracking values across collection APIs, we created special cases of the invoke-virtual instruction for calls to methods on the java/util/ArrayList standard class (including add(), get(), and toArray()), as well as special cases of invoke-direct for two of its constructors. Similar to regular Java arrays, we treated ArrayLists as unordered collections. Combined with tracking of the return value of the android/database/Cursor;->getString() method which is used, among other things, to retrieve phone numbers of contacts, we were able to verify for an app that only contact phone numbers were used as the destination argument of the android/telephony/SmsManager;->sendTextMessage() method.

A typical pattern for malware is to send messages to hardcoded numbers. We found an app in our data set for which the analysis determined that the string “1277” is the only possible value given as the destination for text messages. The number is a Danish premium number but the description of the app on Google Play did state that the app sends a premium message for each look-up, so the app cannot be classified as malicious even though it seems to make money on inattentive users.

Some apps that use hardcoded numbers (whether malicious or not) are specialised and store different numbers for different countries in XML files. These require a considerable number of API calls to parse and extract, so in general finding this type of malware requires implementing more API methods. We also note that the above result in which only phone numbers of contacts are used is not enough, on its own, to classify an app as benign. A malicious app could for example covertly create new contact entries with expensive numbers beforehand. However, some malware authors are looking for financial gain from a minimal coding effort. In the next section we present such a case.

8.1. The FakePlayer Trojan

We also analysed a known malicious app posing as a movie player [3], named Android/FakePlayerA and AndroidOS.FakePlayer by anti-virus vendors. It is a very small app with 15 methods in nine classes. Five of the classes are standard Android "R" resource classes with a constructor each. Running the analysis takes 0.3 s and less than 10 MB of memory. It identifies 45 method calls comprising 27 unresolved API calls, six calls to the java/lang/Object constructor API method, six calls within the app, hereof four unique, and, finally, six calls to the sendTextMessage() API method, hereof two unique. Fig. 4 shows a portion of the call graph. The unresolved part includes calls to the Android android/database/Cursor;->getString() method which is used, among other things, to retrieve phone numbers of contacts, we were able to verify for an app that only contact phone numbers were used as the destination argument of the android/telephony/SmsManager;->sendTextMessage() method.

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useful, necessary even, for developing new analyses, designing heuristics to cover advanced language system features, and for prioritising which among the many language features, libraries, and components in an Android system to focus on in future work.

The main insights revealed by our study was: (1) that almost all instructions in the Dalvik instruction set are used in most apps; (2) use of advanced language/system features, e.g., reflection, dynamic class loading etc. is widespread.

Based on the app study, we have developed a formal operational semantics for the core Dalvik bytecode language, incorporating all major core language features, including dynamic dispatch, exceptions, and reflection. Concurrency is the most notable omission and is the topic of future work. Of the non-core language features that have not been included in the formalisation, libraries are the most important. Without a good model of libraries and their APIs, it is almost impossible to perform analysis of real-life apps. The formal semantics is, of course, a prerequisite for formally developing a sound program analysis and the basis for a proof of correctness. However, a formal semantics is also useful as succinct yet detailed and unambiguous documentation of a language and, in particular, of advanced and subtle language features. In our semantics, this is especially evident in the rules dealing with reflection. We believe that our semantics can serve as a useful tread stone for the Dalvik community, both for developing and discussing (new) language features, but also for better tool support able to handle also corner cases.

In this paper, the formal semantics was used for designing and formally specifying, in a very direct way, a sound control flow analysis. As was the case for the semantics, we believe that the formal specification of a fundamental analysis, such as the control flow analysis, is useful for a wider community for understanding the issues involved in analysing Dalvik bytecode. In addition, many advanced analyses, e.g., secure information flow analysis, use control flow analysis as a building block. Providing this building block, backed by a formal semantics, significantly simplifies the development of new, advanced analyses. During the development of the flow logic specification, the main difficulties turned out to be, again, the reflection features as well as subtle interactions between apps and the Android system.

Finally, a prototype implementation of the analysis was developed by systematically, and almost mechanically, turning the flow logic specification for a program under analysis into a corresponding Prolog program. By executing and querying the Prolog program the analysis result can be established. The prototype was mainly developed as a testbed for variations of and extensions to the control flow analysis. Furthermore, even though high performance was not a goal of the prototype, working with the prototype helped identify particularly costly parts of the analysis, e.g., that taking all the built-in exceptions into account in the exception analysis is prohibitively expensive.
Future work The work described in this paper leaves many avenues open for future work. However, we have identified the following four areas as most promising: (1) extending the semantics and analysis to handle concurrency; (2) extending the analysis of reflection, e.g., by adding string analysis; (3) formal modelling/specification of the Android system and libraries, e.g., by making formal (abstract) models/specifications for the APIs and use these for analysis; and (4) development of a more comprehensive and efficient implementation, e.g., using Datalog.

Acknowledgements

We would like to thank the anonymous reviewers for catching subtle (and not so subtle) errors and for providing detailed and constructive feedback.

Appendix A. Generalised instruction set

<table>
<thead>
<tr>
<th>New instruction</th>
<th>Opcode</th>
<th>Corresponding original instructions</th>
</tr>
</thead>
<tbody>
<tr>
<td>nop</td>
<td>00</td>
<td>nop</td>
</tr>
<tr>
<td>move</td>
<td>01..09</td>
<td>move, move/from16, move/16, move-wide, ...</td>
</tr>
<tr>
<td>move-result</td>
<td>0a..0c</td>
<td>move-result, move-result-wide, move-result-object</td>
</tr>
<tr>
<td>move-exception</td>
<td>0d</td>
<td>move-exception</td>
</tr>
<tr>
<td>return-void</td>
<td>0e</td>
<td>return-void</td>
</tr>
<tr>
<td>return</td>
<td>0f..11</td>
<td>return, return-wide, return-object</td>
</tr>
<tr>
<td>const</td>
<td>12..19</td>
<td>const/4, const/16, const, const/high16, ...</td>
</tr>
<tr>
<td>const-string</td>
<td>1a..1b</td>
<td>const-string, const-string/jumbo</td>
</tr>
<tr>
<td>const-class</td>
<td>1c</td>
<td>const-class</td>
</tr>
<tr>
<td>monitor-enter</td>
<td>1d</td>
<td>monitor-enter</td>
</tr>
<tr>
<td>monitor-exit</td>
<td>1e</td>
<td>monitor-exit</td>
</tr>
<tr>
<td>check-cast</td>
<td>1f</td>
<td>check-cast</td>
</tr>
<tr>
<td>instance-of</td>
<td>20</td>
<td>instance-of</td>
</tr>
<tr>
<td>array-length</td>
<td>21</td>
<td>array-length</td>
</tr>
<tr>
<td>new-instance</td>
<td>22</td>
<td>new-instance</td>
</tr>
<tr>
<td>new-array</td>
<td>23</td>
<td>new-array</td>
</tr>
<tr>
<td>filled-new-array</td>
<td>24..25</td>
<td>filled-new-array, filled-new-array/range</td>
</tr>
<tr>
<td>fill-array-data</td>
<td>26</td>
<td>fill-array-data</td>
</tr>
<tr>
<td>throw</td>
<td>27</td>
<td>throw</td>
</tr>
<tr>
<td>goto</td>
<td>28..2a</td>
<td>goto, goto/16, goto/32</td>
</tr>
<tr>
<td>packed-switch</td>
<td>2b</td>
<td>packed-switch</td>
</tr>
<tr>
<td>sparse-switch</td>
<td>2c</td>
<td>sparse-switch</td>
</tr>
<tr>
<td>cmp</td>
<td>2d..31</td>
<td>cmpl-float, cmpg-float, cmpl-double, ...</td>
</tr>
<tr>
<td>if</td>
<td>32..37</td>
<td>if-eq, if-ne, if-lt, if-ge, if-gt, if-le</td>
</tr>
<tr>
<td>ifz</td>
<td>38..3d</td>
<td>if-eqz, if-nez, if-ltz, if-gez, if-gtz, if-lez</td>
</tr>
<tr>
<td>aget</td>
<td>44..4a</td>
<td>aget, aget-wide, aget-object, aget-boolean, ...</td>
</tr>
<tr>
<td>aput</td>
<td>4b..51</td>
<td>aput, aput-wide, aput-object, aput-boolean, ...</td>
</tr>
<tr>
<td>iget</td>
<td>52..58</td>
<td>iget, iget-wide, iget-object, iget-boolean, ...</td>
</tr>
<tr>
<td>iput</td>
<td>59..6f</td>
<td>iput, iput-wide, iput-object, iput-boolean, ...</td>
</tr>
<tr>
<td>sget</td>
<td>60..66</td>
<td>sget, sget-wide, sget-object, sget-boolean, ...</td>
</tr>
<tr>
<td>sput</td>
<td>67..6d</td>
<td>sput, sput-wide, sput-object, sput-boolean, ...</td>
</tr>
<tr>
<td>invoke-direct</td>
<td>70, 76</td>
<td>invoke-direct, invoke-direct/range</td>
</tr>
<tr>
<td>invoke-supers</td>
<td>6f, 75</td>
<td>invoke-supers, invoke-supers/range</td>
</tr>
<tr>
<td>invoke-interface</td>
<td>71, 77</td>
<td>invoke-interface, invoke-interface/range</td>
</tr>
<tr>
<td>unop</td>
<td>7b..8f</td>
<td>neg-int, not-int, neg-long, not-long, ...</td>
</tr>
<tr>
<td>binop</td>
<td>90..e2</td>
<td>add-int, sub-int, ...add-int/2addr, ...</td>
</tr>
<tr>
<td>binop-lit</td>
<td>d0..e2</td>
<td>add-int/lit16, rsbub-int, mul-int/lit16, ...</td>
</tr>
</tbody>
</table>

Appendix B. Occurrences of instructions

The generalised instructions in our data set ordered by the percentage of the 1700 apps in our data set that use the instruction.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Occurs in</th>
<th># of occurrences</th>
<th>Part of total occ.</th>
</tr>
</thead>
<tbody>
<tr>
<td>invoke-direct</td>
<td>100.00%</td>
<td>4,533,934</td>
<td>4.80%</td>
</tr>
<tr>
<td>return-void</td>
<td>100.00%</td>
<td>2,683,104</td>
<td>2.84%</td>
</tr>
<tr>
<td>invoke-virtual</td>
<td>99.59%</td>
<td>12,718,970</td>
<td>13.47%</td>
</tr>
<tr>
<td>const</td>
<td>99.53%</td>
<td>8,157,468</td>
<td>8.64%</td>
</tr>
<tr>
<td>move-result</td>
<td>99.47%</td>
<td>12,391,920</td>
<td>13.13%</td>
</tr>
</tbody>
</table>
The type domains are specified using BNF notation:

### Table 1 (continued)

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Occurs in</th>
<th># of occurrences</th>
<th>Part of total occ.</th>
</tr>
</thead>
<tbody>
<tr>
<td>invoke-super</td>
<td>99.47%</td>
<td>215,434</td>
<td>0.23%</td>
</tr>
<tr>
<td>const-string</td>
<td>99.29%</td>
<td>5,200,603</td>
<td>5.51%</td>
</tr>
<tr>
<td>new-instance</td>
<td>99.29%</td>
<td>2,900,269</td>
<td>3.07%</td>
</tr>
<tr>
<td>invoke-static</td>
<td>99.24%</td>
<td>3,833,347</td>
<td>4.06%</td>
</tr>
<tr>
<td>input</td>
<td>99.12%</td>
<td>3,389,122</td>
<td>3.59%</td>
</tr>
<tr>
<td>get</td>
<td>99.06%</td>
<td>8,062,226</td>
<td>8.54%</td>
</tr>
<tr>
<td>if</td>
<td>99.06%</td>
<td>3,984,192</td>
<td>4.22%</td>
</tr>
<tr>
<td>goto</td>
<td>98.76%</td>
<td>3,263,902</td>
<td>3.46%</td>
</tr>
<tr>
<td>return</td>
<td>98.06%</td>
<td>2,166,727</td>
<td>2.29%</td>
</tr>
<tr>
<td>move-exception</td>
<td>97.71%</td>
<td>761,554</td>
<td>0.81%</td>
</tr>
<tr>
<td>check-cast</td>
<td>97.53%</td>
<td>1,055,790</td>
<td>1.12%</td>
</tr>
<tr>
<td>if</td>
<td>97.24%</td>
<td>1,304,228</td>
<td>1.38%</td>
</tr>
<tr>
<td>binop-lit</td>
<td>96.59%</td>
<td>1,232,732</td>
<td>1.31%</td>
</tr>
<tr>
<td>invoke-interface</td>
<td>96.35%</td>
<td>1,761,883</td>
<td>1.87%</td>
</tr>
<tr>
<td>move</td>
<td>96.24%</td>
<td>5,503,780</td>
<td>5.83%</td>
</tr>
<tr>
<td>new-array</td>
<td>95.47%</td>
<td>557,610</td>
<td>0.59%</td>
</tr>
<tr>
<td>sget</td>
<td>95.18%</td>
<td>1,792,583</td>
<td>1.90%</td>
</tr>
<tr>
<td>aput</td>
<td>94.88%</td>
<td>1,864,219</td>
<td>1.97%</td>
</tr>
<tr>
<td>binop</td>
<td>94.53%</td>
<td>1,218,279</td>
<td>1.29%</td>
</tr>
<tr>
<td>aget</td>
<td>94.47%</td>
<td>734,425</td>
<td>0.78%</td>
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<tr>
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<td>22.29%</td>
<td>1930</td>
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</table>

Total       94,413,932 100.00%

**Appendix C. Structural domains**

The semantic domains that were left out of the text are listed here. The domains included in App could be specified further for an extension of the semantics into the Android API.

\[
\text{App} = \langle \text{name} : \text{AppName} \rangle \times \langle \text{classes} : \mathcal{P}(\text{Class}) \rangle \times \\
\langle \text{interfaces} : \mathcal{P}(\text{Interface}) \rangle \times \langle \text{manifest} : \text{Manifest} \rangle \times \\
\langle \text{certificate} : \text{Certificate} \rangle \times \langle \text{resources} : \mathcal{P}(\text{Resource}) \rangle \times \\
\langle \text{assets} : \mathcal{P}(\text{Asset}) \rangle \times \langle \text{libs} : \mathcal{P}(\text{Lib}) \rangle
\]

\[
\text{Package} = \langle \text{name} : \text{PackageName} \rangle \times \langle \text{app} : \text{App} \rangle \times \langle \text{classes} : \mathcal{P}(\text{Class}) \rangle
\]

\[
\text{DataTable} = \text{ArrayData} \cup \text{SparseSwitch} \cup \text{PackedSwitch}
\]

\[
\text{ArrayData} = \langle \text{size} : \mathbb{N}_0 \rangle \times \langle \text{data} : \mathbb{N}_0 \rightarrow \text{Prim} \rangle
\]

\[
\text{SparseSwitch} = \langle \text{sparseTargets} : \mathbb{N}_0 \rightarrow \text{PC} \rangle
\]

\[
\text{PackedSwitch} = \langle \text{firstKey} : \mathbb{N}_0 \rangle \times \langle \text{size} : \mathbb{N}_0 \rangle \times \langle \text{packedTargets} : \mathbb{N}_0 \rightarrow \text{PC} \rangle
\]

The type domains are specified using BNF notation:

\[
\text{Type} ::= \text{RefType} \mid \text{PrimType}
\]

\[
\text{PrimType} ::= \text{PrimSingle} \mid \text{PrimDouble}
\]

\[
\text{PrimSingle} ::= \text{boolean} \mid \text{char} \mid \text{byte} \mid \text{short} \mid \text{int} \mid \text{float}
\]

\[
\text{PrimDouble} ::= \text{long} \mid \text{double}
\]

\[
\text{RefType} ::= \text{Class} \mid \text{ArrayType}
\]

\[
\text{ArrayType} ::= \text{ArrayTypeSingle} \mid \text{ArrayTypeDouble}
\]

\[
\text{ArrayTypeSingle} ::= \text{array} (\text{RefType} \mid \text{PrimSingle})
\]

\[
\text{ArrayTypeDouble} ::= \text{array} \text{PrimDouble}
\]
Class hierarchy functions and the subtyping relation:

\[
\begin{align*}
    \text{super}^*(\bot) &= \emptyset \\
    \text{super}^*(\text{cl}) &= \{\text{cl.super}\} \cup (\text{cl.super}).\text{super}^* \\
    \text{implements}^*(\bot) &= \emptyset \\
    \text{implements}^*(\text{cl}) &= \text{cl.implements} \cup (\text{cl.super}).\text{implements}^* \cup (\text{cl.implements}).\text{super}^* \\
    \text{super}(\text{ifaces}) &= \bigcup_{\text{iface} \in \text{ifaces}} \text{iface.super} \cup (\text{iface.super}).\text{super}^*
\end{align*}
\]

\[
\begin{align*}
    cl \in \text{Class} \\
    cl' \in \text{super}^*(cl) \\
    iface \in \text{implements}^*(cl) \\
    t \leq t' \\
    \text{(array } t) \leq (\text{array } t')
\end{align*}
\]

Appendix D. Subject reduction proof

We here present four cases of the subject reduction proof introduced in Section 4.3. The cases demonstrate updates to a register value and the heap as well as the central `invoke-virtual` instruction and object sharing in the `const-class` instruction.

D.1. The `move` case

By definition:

\[
\begin{align*}
    m.\text{instructionAt}(pc) &= \text{move } v_1 \to v_2 \\
    A \vdash (S, H, \langle m, pc, R \rangle :: SF) \Longrightarrow (S, H, \langle m, pc + 1, R[v_1 \mapsto R(v_2)] \rangle :: SF)
\end{align*}
\]

For the proof we assume an analysis result satisfying the judgement for the instruction parameterised by its program point as well as an instantiation of the theorem assumption with an expansion of the domains to facilitate referencing:

\[
\begin{align*}
    (\hat{S}, \hat{H}, \hat{R}) &\models (m, pc) : \text{move } v_1 \to v_2 \tag{D.1} \\
    (S, H, \langle m, pc, R \rangle :: SF) \mathcal{R}_{\text{Conf}} (\hat{S}, \hat{H}, \hat{R}) \tag{D.2}
\end{align*}
\]

From (D.1) and the flow logic judgement (see Section 4.2) we get

\[
\begin{align*}
    \hat{R}(m, pc)(v_2) &\subseteq \hat{R}(m, pc + 1)(v_1) \tag{D.3} \\
    \hat{R}(m, pc) &\subseteq [v_1] \hat{R}(m, pc + 1) \tag{D.4}
\end{align*}
\]

and from (D.2) it follows that

\[
\begin{align*}
    \mu^H_{\text{StaticHeap}}(S) &\subseteq \hat{S} \tag{D.5} \\
    \mu_{\text{Heap}}(H) &\subseteq \hat{H} \tag{D.6} \\
    \mu^H_{\text{LocalReg}}(R) &\subseteq \hat{R}(m, pc) \tag{D.7}
\end{align*}
\]

From (D.7) and the definition of \(\subseteq\) (see Section 4.1) we have

\[
\begin{align*}
    \mu^H_{\text{Val}}(R(v_2)) &\subseteq \hat{R}(m, pc)(v_2) \tag{D.8}
\end{align*}
\]

While (D.8) holds for any \(v_2\) we are interested in the specific \(v_2\) given as an argument to the instruction. Combining (D.8) and (D.3),

\[
\begin{align*}
    \mu^H_{\text{Val}}(R(v_2)) &\subseteq \hat{R}(m, pc)(v_2) \subseteq \hat{R}(m, pc + 1)(v_1) \tag{D.9}
\end{align*}
\]

From (D.4) we get

\[
\begin{align*}
    \forall r \in \text{dom}(\hat{R}(m, pc) \setminus \{v_1\}) : \hat{R}(m, pc)(r) &\subseteq \hat{R}(m, pc + 1)(r) \tag{D.10}
\end{align*}
\]

Now (D.9) covers the case where we consider register \(v_1\) and (D.10) covers all other cases. Therefore:

\[
\mu^H_{\text{LocalReg}}(R[v_1 \mapsto R(v_2)]) \subseteq \hat{R}(m, pc + 1) \tag{D.11}
\]

The case then follows from (D.5), (D.6), and (D.11).
D.2. The `iput` case

By definition:

\[
\begin{align*}
    m\text{-instructionAt}(pc) &= \text{iput } v_1 \ v_2 \ fld \\
    R(v_2) &= \text{loc } \neq \text{null } \quad o = H(\text{loc}) \\
    \alpha \text{ class}\langle \text{fld.class} \quad o' = o[\text{field} \mapsto o[\text{field} \mapsto R(v_1)]] \quad \\
    A \vdash \langle S, H, \langle m, pc, R \rangle :: SF \rangle \implies \langle S, H[\text{loc} \mapsto o'], \langle m, pc + 1, R \rangle :: SF \rangle
\end{align*}
\]

We assume

\[
\begin{align*}
    \langle \hat{S}, \hat{H}, \hat{R} \rangle &= (m, pc) : \text{iput } v_1 \ v_2 \ fld \\
    \langle S, H, \langle m, pc, R \rangle :: SF \rangle \implies \hat{S}, \hat{H}, \hat{R}
\end{align*}
\] (D.12)

From (D.12) we get

\[
\hat{R}(m, pc) \subseteq \hat{R}(m, pc + 1)
\] (D.14)

and

\[
\forall (\text{ObjRef } cl, m', pc') \in \hat{R}(m, pc)(v_2) : \\
    cl \leq \text{fld.class} \Rightarrow \\
    \hat{R}(m, pc)(v_1) \subseteq \hat{H}(\text{ObjRef } cl, m', pc')(fld)
\] (D.15)

From (D.13) follows

\[
\beta^H_{\text{StaticHeap}}(S) \subseteq \hat{S}
\] (D.16)

\[
\beta_{\text{Heap}}(H) \subseteq \hat{H}
\] (D.17)

\[
\beta^H_{\text{LocalReg}}(R) \subseteq \hat{R}(m, pc)
\] (D.18)

Combining (D.18) and (D.14),

\[
\beta^H_{\text{LocalReg}}(R) \subseteq \hat{R}(m, pc) \subseteq \hat{R}(m, pc + 1)
\] (D.19)

Two out of the three parts of the correctness of the new configuration follow from (D.16) and (D.19).

For the final part, the dynamic heap, we recall from Section 4.3 that the abstract representation of the concrete heap is the union of the representations of its objects who are in turn represented as the representations of their fields. Since we know from (D.17) that the existing representation is part of the analysis result, we can reduce the problem of showing

\[
\beta_{\text{Heap}}(H[\text{loc} \mapsto o[\text{field} \mapsto o[\text{field} \mapsto R(v_1)]]]) \subseteq \hat{H}
\] (D.20)

to showing

\[
\beta^H_{\text{Val}}(R(v_1)) \subseteq \hat{H}(\text{ObjRef } cl, m', pc')(fld)
\] (D.21)

From (D.18) we have

\[
\beta^H_{\text{Val}}(R(v_1)) \subseteq \hat{R}(m, pc)(v_1)
\] (D.22)

Because the semantic step has been taken, the premises of the semantic rule must have been satisfied, so from (D.15) we get

\[
\hat{R}(m, pc)(v_1) \subseteq \hat{H}(\text{ObjRef } cl, m', pc')(fld)
\] (D.23)

Combining (D.22) and (D.23) finishes the proof of this case.

D.3. The `const-class sharing object case`

There are two sub-cases for `const-class`: one that creates a new object and one that re-uses and shares an already existing object. Here we look only at the latter case. By definition:

\[
\begin{align*}
    m\text{-instructionAt}(pc) &= \text{const-class } \nu \ cl \\
    \text{findClassObject}(H, cl) &= \text{loc.c} \neq \bot \\
    A \vdash \langle S, H, \langle m, pc, R \rangle :: SF \rangle \implies \langle S, H, \langle m, pc + 1, R[v \mapsto \text{loc.c}] \rangle :: SF \rangle
\end{align*}
\]

We assume
From (D.33) follows

\[(S, H, R) \not\models SF \quad \mathcal{R}_\text{Conf} (\hat{S}, \hat{H}, \hat{R})\]  

(D.24)

From (D.32) we get

\[\beta_{\text{StaticHeap}}(S) \subseteq \hat{S}\]  

(D.26)

\[\beta_{\text{Heap}}(H) \subseteq \hat{H}\]  

(D.27)

\[\beta_{\text{LocalReg}}(R) \subseteq \hat{R}(m, pc)\]  

(D.28)

Further, it follows from (D.24) that

\[\forall v' : v \neq v' \Rightarrow \hat{R}(m, pc + 1)(v') = \hat{R}(m, pc)(v')\]  

(D.29)

and

\[\text{dom}(\hat{H})_{\text{java/lang/Class}} \subseteq \hat{R}(m, pc + 1)(v)\]

Now, since \(locc = \text{findClassObject}(H, cl)\) it follows that \(H(locc) \in \text{Object} \) with \(H(locc).\text{class} = \text{java/lang/Class}, H(locc).\text{class} = cl, \) and \(H(locc).\text{origin} = (m', pc')\) for some \((m', pc')\), it follows from (D.27) that:

\[\beta_{\text{Object}}(H(locc)) \subseteq \beta_{\text{Heap}}(H)(\text{ObjRef}(\text{java/lang/Class}, m', pc'))\]

where \(\beta_{\text{VRef}}(locc) = (\text{ObjRef}(\text{java/lang/Class}, m', pc'))\). We can now calculate as follows:

\[(\beta_{\text{VRef}} \circ R[v \mapsto \text{locc}](v) = \beta_{\text{VRef}}(locc)\]

\[\subseteq \text{dom}(\hat{H})_{\text{java/lang/Class}}\]

\[\subseteq \hat{R}(m, pc + 1)(v)\]

Combining (D.29) and (D.30) we have

\[\beta_{\text{LocalReg}}(R[v \mapsto \text{locc}]) \subseteq \hat{R}(m, pc + 1)\]  

(D.31)

The case now follows from (D.26), (D.27), and (D.31).

D.4. The invoke-virtual case

By definition:

\[m.\text{instructionAt}(pc) = \text{invoke-virtual } v_1 \ldots v_n \text{ meth}\]

\[R(v_1) = \text{loc} \neq \text{null} \quad o = H(\text{loc})\]

\[n = \text{arity}(\text{meth}) \quad m' = \text{resolveMethod}(\text{meth, o.class}) \neq \bot\]

\[R' = [0 \mapsto \bot, \ldots, m'.\text{numLocals} - 1 \mapsto \bot, m'.\text{numLocals} \mapsto R(v_1), \ldots, m'.\text{numLocals} + n - 1 \mapsto R(v_n)]\]

\[A \vdash (S, H, (m, pc, R) :: SF) \quad \Rightarrow (S, H, (m', 0, R') :: (m, pc, R) :: SF)\]

We assume

\[(\hat{S}, \hat{H}, \hat{R}) \models (m, pc) : \text{invoke-virtual } v_1 \ldots v_n \text{ meth}\]

\[\langle S, H, (m, pc, R) :: SF \rangle \quad \mathcal{R}_\text{Conf} (\hat{S}, \hat{H}, \hat{R})\]  

(D.32)

From (D.32) we get

\[\forall \text{ObjRef } cl, m_t, pc_t \in \hat{R}(m, pc)(v_1) : \]

\[m' = \text{resolveMethod}(\text{meth, cl})\]

\[\forall 1 \leq i \leq n : \]

\[\hat{R}(m, pc)(v_i) \subseteq \hat{R}(m', 0)(m'.\text{numLocals} - 1 + i)\]  

(D.34)

From (D.33) follows

\[\beta_{\text{StaticHeap}}(S) \subseteq \hat{S}\]  

(D.35)

\[\beta_{\text{Heap}}(H) \subseteq \hat{H}\]  

(D.36)

\[\beta_{\text{LocalReg}}(R) \subseteq \hat{R}(m, pc)\]  

(D.37)
It remains to be shown that the new stack frame, \((m', 0, R')\), is correctly represented in \(\hat{R}\). It follows from (D.33) that the rest of the stack, \((m, pc, R) :: S\), is correctly represented. Therefore we now need to show

\[
\beta^H_{\text{LocalReg}}(R') \sqsubseteq \hat{R}(m', 0) \tag{D.38}
\]

and that the \(m'\) referenced in the analysis is the same as the one in the semantics. The latter follows because \(\hat{R}(m, pc)(v_1)\) is a sound over-approximation of \(R(v_1)\) and thus, by the induction hypothesis, must contain an abstract representation of the correct object reference. The same method lookup is used in the semantics and in (D.34) so the correct method must be included in the analysis.

By the definition of \(\sqsubseteq\), (D.38) is equivalent to

\[
\forall r \in \text{dom}(R') : \beta^H_{\text{Val}}(R'(r)) \sqsubseteq \hat{R}(m', 0)(r) \tag{D.39}
\]

Since \(R'\) is used both to transfer parameter values to the called method as well as for storing (the value of) local variables in the called method, we split this into two cases for \(r \in \text{dom}(R'):\)

1. \(0 \leq r \leq m'.\text{numLocals} - 1\)
2. \(m'.\text{numLocals} \leq r \leq m'.\text{numLocals} + n - 1\)

From the semantics it follows that \(R'(r) = \bot_{\text{Val}}\) for all \(r\) such that \(0 \leq r \leq m'.\text{numLocals} - 1\) and thus

\[
\beta^H_{\text{LocalReg}}(R')(r) = \beta^H_{\text{Val}}(R'(r))
\]

\[
= \beta^H_{\text{Val}}(\bot_{\text{Val}})
\]

\[
= \bot_{\text{Val}}
\]

\[
\sqsubseteq \hat{R}(m', 0)(r) \tag{D.40}
\]

which completes the first sub-case. For the second sub-case we have from the definition of \(R'\) in the semantics and Eqs. (D.37) and (D.34) that for \(1 \leq i \leq n:\)

\[
\beta^H_{\text{Val}}(R'(m'.\text{numLocals} - 1 + i)) = \beta^H_{\text{Val}}(R(v_i))
\]

\[
\sqsubseteq \hat{R}(m, pc)(v_i)
\]

\[
\sqsubseteq \hat{R}(m, 0)(m'.\text{numLocals} - 1 + i) \tag{D.41}
\]

Combining Eqs. (D.40) and (D.41) we conclude that \(\beta^H_{\text{LocalReg}}(R') \sqsubseteq \hat{R}(m', 0)\) which finishes the proof.

References


